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ANALYSIS AND OPTIMIZATION OF ELASTIC-PLASTIC FRAMING STRUCTURES UNDER COMPLEX CONSTRAINTS

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Abstract. The paper presents a mathematical model for bar cross-sectional optimization of steel structureunder strength, stiffness and stability constraints. The theory of mathematical programming of extremum energy principles has been used for developing the introduced model. Solving a non-linear mathematical problem is subject to the *MatLab* programming environment. Because of the existing relationship between elastic response values and the optimized parameters of the structure, the problem has been calculated iteratively. The calculation algorithm has been applied to a frame with a truss span. The framing structure has been discretized by finite bar elements. The minimum volume of the structure that has not reached full plastic collapse but its individual members have experienced plastic deformation has been designed. According to the obtained optimal project, standard tube profiles have been chosen.

Keywords: optimization, elastic-plastic framing structure, energy principles, strength, stiffness, stability, *MatLab*.

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Introduction

Optimal design is aimed at planning a project of the construction that ensures the strength, stiffness and stability of the designed structure as well as requires minimum cost to produce and operate the formation (Čyras *et al.* 2004).

Over the past decades, the structure optimization theory, methods, calculation algorithms and their interface with computer simulation and design programs have been improved. It has been discovered that one of the most effective methods of structure optimization is the application of both – the theory of mathematical programming of extremum energy principles and plastic properties of materials (Atkočiūnas, Karkauskas 2010; Karkauskas, Norkus 2006; Popov, Karkauskas 2005; Kalanta 1997). It is also obvious that the evaluation of deformable state parameters and plastic properties of materials accompanies more expressed work on the structure at different loading stages (Kaliszky, Logo 2002; Romero *et al.* 2004). This calculation method helps with the economical exploitation of materials and creates a much more rational design project (Atkočiūnas, Karkauskas 2010; Kalanta 1997).

The minimal material mass (volume) of the structure is one of the main criteria of optimality and is being applied to the problems of construction optimization (Makris, Provatidis 2002; Luh, Lin 2008; Hernandez *et al.* 2005; Pereyra *et al.* 2003; Gil, Andreu 2001). In most cases, extremum energy principles are formulated for identifying the actual stress and strain state of such formation. When static (internal forces or stresses) or kinematic (displacements or deforma-

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tions) variables are chosen as the main unknowns, extremum problems are obtained. The problems consist of an objective function and constraints describing the actual state of stress and strain. When these problems are resolved, the actual state of the stress and strain of the structure as well as its optimal parameters are identified (Atkočiūnas, Karkauskas 2010; Karkauskas, Norkus 2006).

The main tool for optimizing problems is the limit equilibrium theory ensuring the strength of the structure. However, often an optimal structure in terms of strength may not meet maintenance requirements. It happens when the structure that has not reached full plastic collapse faces high plastic deformations and displacements. In engineering practice, it is referred to as a limit state. There are two limit states of plastically deformed structures – the limit state of safety related to plastic collapse and the limit state of eligibility related to limit deformations (STR 2.05.08:2005; Eurokodas 3).

The listed reasons complicate the practical application of the limit equilibrium theory; however, to avoid this, deformability constraints could be involved into the mathematical models of optimization problems. The evaluation of the parameters of the deformed state of the structure allows restricting the displacements of individual nodes or elements, limited slenderness of structure elements, etc.

The research is aimed at:

- forming and improving optimization problems calculating algorithms in the *MatLab* environment under the evaluation of the strength, stiffness and stability constraints of the framing structure and with regard to non-elastic steel characteristics;
- numerical experiment on the analysis and optimization of the elastic-plastic framing structure under displacement constraints and at evaluating the stability of the bars under compression.

The methodology is illustrated by a light-type framing structure with a truss span under quasi-static, single type loading (Pedersen, Nielsen 2001). The presented framing structure is designed from tube profiles having functional dependence between cross-section parameters, type and the thickness of the web. Optimality criterion for bar cross-sections is the used volume of the material.

1. Mathematical model for framing structure optimization in plastic collapse

The optimization problem of the plastic collapse of the steel framing structure already having plastic deformations is formulated when the configuration of the structure, external loads, its adding location, direction, value and optimality criterion are known. According to the optimum criterion, optimal resistant (limit) internal forces $\boldsymbol{S}_0$ (cross-sectional area A) are found.

Strength, stiffness and stability constraints must comply with the actual stress and strain state of the structure prior to its plastic collapse. Thus, combining all these limitations and the application of the expression of energy dissipation (scattering) as a minimum objective function assists in obtaining a mathematical model for the optimization problem in plastic collapse (Atkočiūnas, Karkauskas 2010):

$$\left.\begin{array}{ll}\text{find} & \boldsymbol{L}^T\boldsymbol{S}_0 \to \min,\\ \text{subject to} & [\Gamma]\boldsymbol{S}_0 - [\Phi]\boldsymbol{S} \ge \boldsymbol{0},\\ & [A]\boldsymbol{S} = \boldsymbol{F},\\ & \boldsymbol{S}_0 \ge \boldsymbol{S}_0^{\min}.\end{array}\right\}, \tag{1}$$

where: $\boldsymbol{L}$ is a vector of element lengths; $\boldsymbol{S}_0$ is a vector of limited internal forces; $[\Gamma]$ is the construction configuration matrix; $[\Phi]$is the matrix of yield conditions for the frame; $\boldsymbol{S}$ is a vector of total internal forces; $[A]$ is the coefficient matrix of structure equilibrium conditions; $\boldsymbol{F}$ is a vector of external forces; $\boldsymbol{S}_0^{\min}$ is a vector of lower restriction bounds to limited internal forces.

When solving the above introduced mathematical programming problem (1), unknowns $\boldsymbol{S}$ and $\boldsymbol{S}_0$ are obtained.

2. The problem analysing an optimal framing structure

A system in plastic collapse is in the limit equilibrium state. Then, any small increase in load leads to the unlimited values of displacements and strains, and therefore is hard to decide on the critical state of the absolute values of displacements and deformations. Because of a plastic mechanism, different deformed body situations may be obtained (Atkočiūnas, Karkauskas 2010).

Knowing the physical parameters and external load of the structure, the problem of analysis occurs, which means that the actual stress and strain state prior to the plastic collapse of the system is searched. The optimal parameters of cross-sections are expressed via limit internal forces $\boldsymbol{S}_0$.

The mathematical model of the task for analysing an optimal framing structure isas follows (Atkočiūnas, Karkauskas 2010):

$$\left.\begin{array}{ll} \text{Find} & \frac{1}{2}\boldsymbol{S}_r^T D\boldsymbol{S}_r \to \min, \\ \text{subject to} & -[\Phi]\boldsymbol{S}_r \geq [\Phi]\boldsymbol{S}_e - [\Gamma]\boldsymbol{S}_0, \\ & [A]\boldsymbol{S}_r = \boldsymbol{0}. \end{array}\right\}, \quad (2)$$

where: $\boldsymbol{S}_r$ is a vector of residual internal forces; D is the general *Hooke's* law flexibility matrix of the initial unstrained construction; $\boldsymbol{S}_e$ is a vector of elastic internal forces.

This is a convex quadratic programming problem (Bazaraa *et al.* 2006). A solution to dealing with the problem of analysis points to residual internal forces $\boldsymbol{S}_r$. Also, actual internal forces $\boldsymbol{S} = \boldsymbol{S}_r + \boldsymbol{S}_e$ and displacements $\boldsymbol{u} = \boldsymbol{u}_r + \boldsymbol{u}_e$ are obtained. Thus, this is the actual stress and strain state of the structure.

Residual displacements $\boldsymbol{u}_r$ are obtained by solving a dual formulation of the problem (2). Moreover, as *MatLab* was used for reaching a solution to the optimization problem, it makes possible to obtain the unknown quantities of dual formulation by dealing with the problem (2).

3. Optimization of elastic-plastic framing structures under displacements constraints

The optimization problem of limiting displacements is formulated in case external loads and its adding location as well as the direction and value affecting the configuration of structure are known. Upper and lower displacement changes are verified. According to structure requirements, the elements that are lower the boundary changes of internal forces are validated (form of optimality criterion form and its coefficients).

Boundary distribution of internal forces should be found, which corresponds to the minimum capacity of the structure that did not reach requirements for plastic collapse.

A proposal to introduce the following boundary restrictions of eligibility (Karkauskas, Nagevičius 2007; Karkauskas, Norkus 2006) embraces

- constraints of maintenance defining conditions that describe the real stress and deformation state of the structure. This is found based on the additional energy of deformation by the minimum principle formalized from a pair of dual extremum problems and *Kuhn* and *Tucker* conditions (Bazaraa *et al.* 2006);
- deformation constraints defining conditions for displacement restriction in particular places of the structure in certain directions:

$$\boldsymbol{u}^- \leq (\boldsymbol{u}_r + \boldsymbol{u}_e) \leq \boldsymbol{u}^+, \quad (3)$$

  where: $\boldsymbol{u}^+ > 0$ and $\boldsymbol{u}^- < 0$ – normative values of upper and lower limits to displacements. By solving the problem analyzing load-displacement dependency, the values of upper and lower limits to displacements are obtained (Atkočiūnas, Karkauskas 2010).
- defined technological or constructive requirements in normative documents for element stability or moments of boundary flexibility with regards to a lower limit to change $\boldsymbol{S}_0^{\min}$.

Then, the total framing structures optimization problem by limiting displacements in the mathematical model that consists of three optimization problems, including plastic collapse, analysis and load displacement dependency (Atkočiūnas, Karkauskas 2010), is written in the following way (Atkočiūnas, Karkauskas 2010):

$$\left.\begin{array}{ll} \text{Find} & \boldsymbol{L}^T\boldsymbol{S}_0 \to \min, \\ \text{subject to} & [\Gamma]\boldsymbol{S}_0 - [\Phi]\boldsymbol{S}_r \geq [\Phi]\boldsymbol{S}_e, \\ & [A]\boldsymbol{S}_r = \boldsymbol{0}, \\ & D\boldsymbol{S}_r - [\Phi]^T\boldsymbol{\lambda} - [A]^T\boldsymbol{u}_r = \boldsymbol{0}, \\ & \boldsymbol{\lambda}^T([\Gamma]\boldsymbol{S}_0 - [\Phi](\boldsymbol{S}_r{}^T\boldsymbol{\lambda} + \boldsymbol{S}_e)) = \boldsymbol{0}, \\ & \boldsymbol{\lambda} \geq \boldsymbol{0}, \\ & [P](\boldsymbol{u}_r + \boldsymbol{u}_e) \leq \boldsymbol{u}^+, \\ & [P](\boldsymbol{u}_r + \boldsymbol{u}_e) \geq \boldsymbol{u}^-, \\ & \boldsymbol{S}_0 \geq \boldsymbol{S}_0^{\min}, \\ & \boldsymbol{u}_e = K^{-1}\boldsymbol{F}, \\ & \boldsymbol{S}_e = D^{-1}[A]^T K^{-1}\boldsymbol{F}. \end{array}\right\}, \quad (4)$$

where: $[P]$ is a logical matrix of the displacement that evaluates lower conditions.

This is a nonlinear mathematical programming problem solved by the iteration method. The fourth restriction condition – *Kuhn* and *Tucker* complementary condition – gives much extremeness and greater complicates a solution to the problem. Thus, this model could be modified by eliminating residual internal forces $\boldsymbol{S}_r$ and residual displacements $\boldsymbol{u}_r$ (Atkočiūnas, Karkauskas 2010).

4. Numerical realization of the optimization problem

For proposing a solution to the iteration process optimizing the problem of analysis, a flat framing structure has been chosen. A numerical experiment has been performed thus including a mathematical model consisting of strength, stiffness and stability constraints.

The configuration and load character of the analyzed structure are presented in the calculation scheme (Fig. 1.).

Strulcural elements are designed considering the cross-sectional profiles of a hot-rolled tube.

The frame is designed with the same limited internal forces for the particular groups of elements of a structure: M_{01} – limited internal force depending on columns, M_{02} – boundary bars of the upper lane of the truss, M_{03} – rest elements of the upper lane of the truss (upper lane of the truss is accepted as uncut), N_{01} – the lower lane of the truss, N_{02} – the grid of the truss (Fig. 2).

The initial geometric characteristics of the cross-section of frame bars are calculated according to the formulas that express functional dependence between cross-sectional parameters (Atkočiūnas, Karkauskas 2010).

$$I_x = a_1 A^{b_1}; \tag{5}$$

$$W_{pl} = a_3 A^{b_3}, \tag{6}$$

where: I_x is a cross-sectional moment of inertia; W_{pl} is a plastic resistance moment of the cross-section; A is the initial cross-sectional area; a_1, b_1, a_3, b_3 are the coefficients of the geometric characteristics of the cross-section that depends on the type of the cross-section and web thickness. The coefficients are given in Table 1.

Steel yield strength for columns and the upper lane of the truss is 275 MPa while the lower lane of

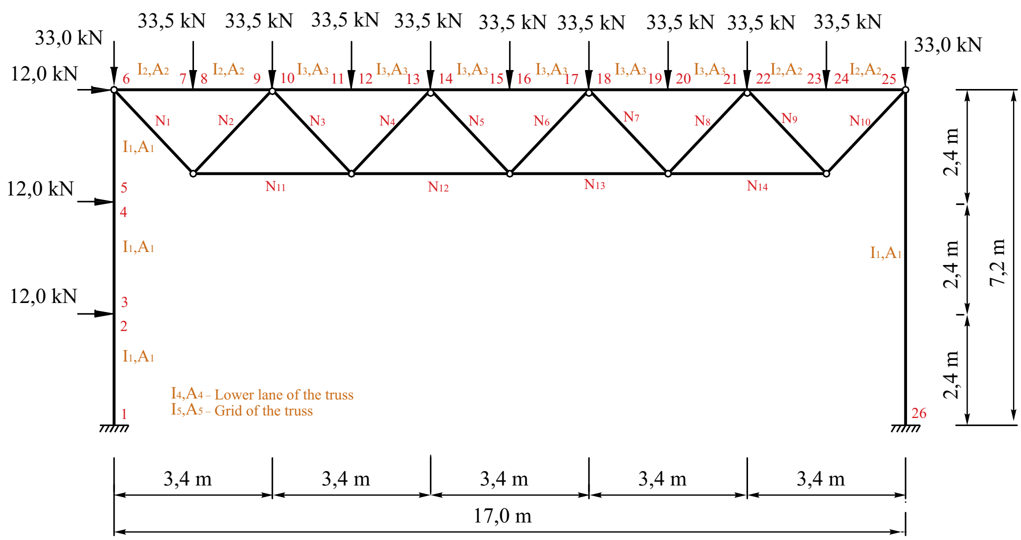

Fig. 1. Computational scheme for a steel framing structure

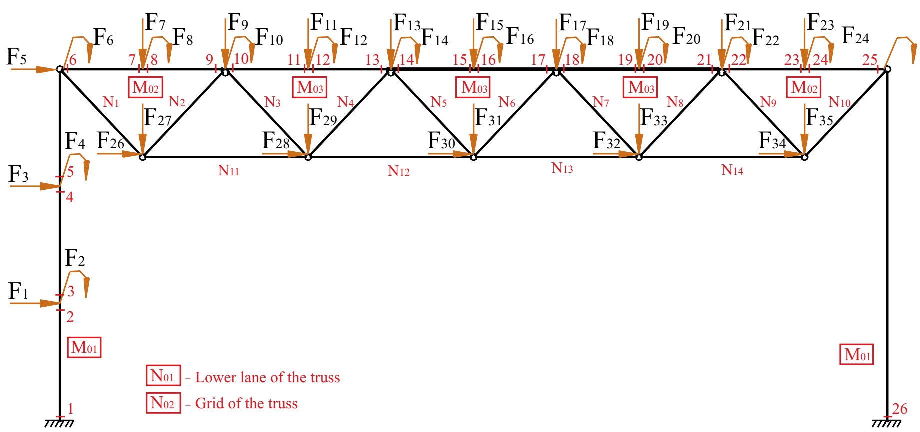

Fig. 2. Discrete model for a steel framing structure

the truss and the grid of the truss makes 235 MPa. Material elasticity (*Young's*) modulus E = 210 GPa.

A discrete model containing 28 finite elements is presented in Figure 2. DOF = 35. All elements of the discrete model are formed as beam-column elements.

As the discrete model of the structure should be adequate for the work of internal forces, columns and the upper lane of the truss are approximated only as bending elements while the lower lane of the truss and the grid of the truss – as tensile or compressive elements. Thus, the unknown number of the internal forces of construction are n = 40.

In the case of bending elements, only the work of those is evaluated and yield conditions are written in the following way:

$$\left.\begin{aligned} M_j &\le M_{0j}, \\ -M_j &\le M_{0j}, \end{aligned}\right\} j = 1,2,...,n. \qquad (7)$$

In the event of elements under tension and compression, only the work of axial forces is evaluated and yield conditions are written in the following way:

$$\left.\begin{aligned} N_j &\le N_{0j}, \\ -N_j &\le N_{crj}, \end{aligned}\right\} j = 1,2,...,n. \qquad (8)$$

Table 1. The coefficients of geometric characteristics of profiles

Type of a cross-section	Web thickness, mm	a_1	b_1,	a_3,	b_3
Rectangular	5	0,0668	2,9665	0,2303	1,9851
Rectangular	6,3	0,0512	2,9099	0,1979	1,9619
Rectangular	8	0,0286	2,9470	0,1488	1,9779
Rectangular	10	0,0185	2,9558	0,1227	1,9782

5. Introduction into restrictions

– *Selecting lower limits to the moments of boundary flexibility*

These boundaries are determined by solving the problem of frame optimization during plastic collapse based on the mathematical model (1) in which internal boundary forces $\boldsymbol{S}_{0,cr}^{\min}$ of construction restrictions are related to the vector of frame elements from the minimum values of the moments of boundary flexibility $\boldsymbol{M}_{0,cr}^{\min}$. Those are calculated based on the formula (Karkauskas, Nagevičius 2007) introducing conditions for requirements for boundary slenderness (STR 2.05.08:2005):

$$M_{0,cr}^{\min} = \sigma_y a_3 \left(\frac{l_b^2}{a_1 \lambda_{\lim}^2} \right)^{\frac{b_3}{b_1 - 1}}, \qquad (9)$$

where: l_b is the buckling length of the element and $\lambda_{\lim}$ is the boundary slenderness of the element selected according to STR 2.05.08:2005 instructions.

The calculated upper lane boundary slenderness of columns and truss – $\lambda_{\lim}$ = 150.

– *Determining internal boundary forces of a compressed bar*

When a bar is affected by deformation, its internal boundary force N_0 is axis force to buckling N_{cr} and is expressed in the following way (STR 2.05.08:2005):

$$N_{cr} = \chi \sigma_y A = \sigma_{cr} A, \qquad (10)$$

where: $\sigma_{cr} = \chi \sigma_y$ is calculated buckling stresses and χ is the reduction coefficient that depends on the non-dimensional slenderness of the bar determined as follows:

$$\bar{\lambda} = \left(\frac{\lambda}{\lambda_E} \right) = \frac{l_b}{i} \frac{1}{\lambda_E} = \frac{l_b}{\sqrt{I_x / A}} \frac{1}{\pi \sqrt{E/\sigma_y}}, \qquad (11)$$

where: λ is bar slenderness, $\lambda_E = \pi \sqrt{E/\sigma_y}$, – *Euler's* slenderness.

Then, the reduction coefficient can be calculated by *EN3* given formula:

$$\chi = \frac{1}{\varphi + \sqrt{\varphi^2 - \bar{\lambda}^2}}, \quad \text{but} \quad \chi \le 1; \qquad (12)$$

$$\varphi = 0,5 \left[1 + \alpha \left(\bar{\lambda} - 0,2 \right) + \bar{\lambda}^2 \right], \qquad (13)$$

where: α = 0.21 for hot forming pipes.

Solving the optimization problem by the mathematical model (1) got the final lower limit vector of internal boundary forces:

$$\boldsymbol{S}_0^{\min} = \boldsymbol{S}_{0,cr}^{\min} = \left[\boldsymbol{M}_{01,cr}^{\min} \ \boldsymbol{M}_{02,cr}^{\min} \ \boldsymbol{M}_{03,cr}^{\min} \ \boldsymbol{N}_{01}^{\min} \ \boldsymbol{N}_{02,cr}^{\min} \right]^T = \left[199,42 \ \ 46,52 \ \ 427,46 \ \ 400,0 \ \ 406,25 \right]^T.$$

– *Selecting the upper and lower limits of displacements*

For this purpose, the calculation of the problem analysing the stress deformed state of the frame is executed employing the mathematical model of the problem analysing load and displacement dependency (Atkočiūnas, Karkauskas 2010). The earlier found values of internal forces $\boldsymbol{S}_0$ and reduced load $\tilde{\boldsymbol{F}} = \gamma_{red} \boldsymbol{F}$ are used for restrictions on the problem. γ_{red} is the factor of reduced load making less than 1.

Frame limits $\boldsymbol{u}_{\max}$ are determined by solving the problem of analysis, i.e. displacement values before plastic collapse $\boldsymbol{u}_{\max}^+$, if $\boldsymbol{u}_{\max}^+ > 0$ or $\boldsymbol{u}_{\max}^-$, if $\boldsymbol{u}_{\max}^- < 0$.

Then, by decreasing reduction factor γ_{red} displacement values $\boldsymbol{u}_{\min}$ are calculated when the first plastic hinge is opened. $\boldsymbol{u}_{\min}$ corresponds to the values of elastic response $\boldsymbol{u}^{+}_{\min}$ if $\boldsymbol{u}^{+}_{\min} > 0$ or $\boldsymbol{u}^{-}_{\min}$ if $\boldsymbol{u}^{-}_{\min} < 0$.

Thus, limits to changes, under conditions of restrictions on the displacements of the mathematical model of the problem should match these values:

$$\boldsymbol{u}^{+} > 0, \text{ tai } \boldsymbol{u}^{+}_{\min} \le \boldsymbol{u}^{+} \le \boldsymbol{u}^{+}_{\max},$$
$$\boldsymbol{u}^{-} < 0, \text{ tai } \boldsymbol{u}^{-}_{\min} \ge \boldsymbol{u}^{-} \ge \boldsymbol{u}^{-}_{\max}. \quad (14)$$

The above provided conditions ensure plastic construction performance and allow optimizing it applying the mathematical model (3). Otherwise, no solution for the problem can be suggested.

Restricted characteristic incision displacements include a top horizontal frame displacement with the boundary value of 4 cm ($u_5^{+} = 0{,}04$, $u_5^{-} = 0$) and vertical truss deflection with the boundary value of 2 cm ($u_{15}^{+} = 0{,}02$, $u_{15}^{-} = 0$).

Having a vector of lower limits from internal forces and load reduction factor γ_{red} = 0.9999, the problem analysing load-displacement dependence can be solved (Atkočiūnas, Karkauskas 2010). With reference to this factor of reduction, upper limits to frame displacements are as follows:

horizontal direction – on the top of the column:

$$\boldsymbol{u}^{+}_{5,\max} = \boldsymbol{u}_{5,e} + \boldsymbol{u}_{5,r} = 5{,}19 + 0{,}00 = 5{,}19 \text{ cm};$$

vertical direction – in the middle of the truss:

$$\boldsymbol{u}^{+}_{15,\max} = \boldsymbol{u}_{15,e} + \boldsymbol{u}_{15,r} = 1{,}21 + 5{,}46 = 6{,}67 \text{ cm}.$$

The load factor is decreased to γ_{red} = 0.7783 when the first plastic hinge is opened in the bars and displacement values:

horizontal direction – on the top of the column:

$$\boldsymbol{u}^{+}_{5,\min} = \boldsymbol{u}_{5,e} + \boldsymbol{u}_{5,r} = 4{,}00 + 0{,}00 = 4{,}00 \text{ cm};$$

vertical direction – in the middle of the truss:

$$\boldsymbol{u}^{+}_{15,\min} = u_{15,e} + u_{15,r} = 0{,}90 + 0{,}00 = 0{,}90 \text{ cm}.$$

Limits to displacement changes correspond to condition (14) and the performance of elastic-plastic construction is ensured.

For the optimization process, starting boundary values of internal forces 20% greater than $\boldsymbol{S}_0^{\min}$ are chosen

$$\boldsymbol{S}_0^{pr} = \left[220{,}0 \;\; 80{,}0 \;\; 450{,}0 \;\; 450{,}0 \;\; 450{,}0\right]^T.$$

By solving the problem of analysis using the initial vector of internal boundary forces, we can state that the initial restriction displacement values match the required conditions (14) because:

$$4{,}00 < u_5^{pr} = 4{,}48 < 5{,}19;$$
$$0{,}90 < u_{15}^{pr} = 3{,}94 < 6{,}67.$$

Results of counting analysis. An optimal project has been achieved in 5 iterations when displacements are restricted and deformed bar stability is evaluated. In total, 2 plastic hinges, including those having 12 and 13 elements in the lower lane of the truss, are obtained. Changes in the dynamics of the projected parameters during iteration processing are given in Table 2. The zero line displays the internal forces of starting boundary (kNm, kN) and the values of the objective function (kNm^2). The project of the optimal frame is shown in the last line.

Table 2. Optimization results under stability and displacement constraints

Iteration	M_{01}	M_{02}	M_{03}	N_{01}	N_{02}	$\boldsymbol{L}^{\mathrm{T}}\boldsymbol{S}_0$
0	240,0	80,0	450,0	450,0	450,0	25870
1	199,4	46,5	427,5	459,7	406,2	23876
2	199,4	46,5	427,5	459,8	406,2	23877

Table 3. Chosen data on the framing structure

Element	Profile	A, cm^2	W_{pl}, cm^3	I_y, cm^4	Mass, kg/m
Columns					
M_{01}	RHS 220×120×6.3	38,89	185,85	979,19	31,3
Truss					
Upper lane M_{02}	RHS 300×100×8	72,57	285,25	1224,41	57,0
Upper lane M_{03}	RHS 300×200×8	75,24	574,46	5041,67	59,1
Lower lane N_{01}	RHS 90×90×5	16,36	51,41	192,93	12,8
Grid N_{02}	RHS 50×50×4	6,95	11,73	23,74	5,45

According to the optimally derived plan and resistant internal forces, tube profiles are selected for the bars of the structure: for the columns of the frame structure and the upper lane of the truss – rectangular cross-sections and for the lower lane of the truss and the grid – square cross-sections (Table 3).

It can be noticed that an optimal plan is achieved by few iterations through limiting construction displacements and evaluating stability. This determines limits on the stability of the bars compressed by the structure. Limitations restrict the free spread of the non-elastic deformations of steel thus decreasing the interval from the formation of the first plastic hinge up to the plastic collapse of the structure (Fig. 3).

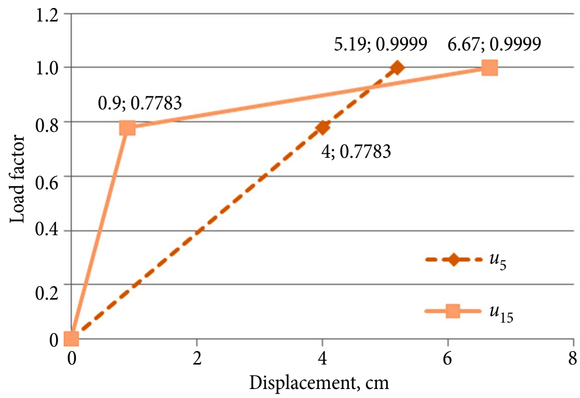

Fig. 3. Diagram of load versus displacements

Table 4. Displacements of the framing structure

Horizontal displacement	$u_{5,max}$ = 3.93 cm
Vertical displacement	$u_{15,max}$ =2.07 cm

Table 4 shows that displacement values meet normative requirements for regulated boundary deflections and displacements for such type of construction, but the performance of construction in the plastic state is less notable. Figure 3 presents the diagram of load-displacement dependency.

Conclusions

1. The algorithm for calculating the optimal project and problems of analysis of the frame structure that has experienced plastic deformations has been made. The non-elastic qualities as well as strength, stiffness and stability of steel structure have been evaluated.
2. Finding an optimal project of the structure is a complex task due to the existing relationship between elastic response values and optimized parameters which therefore determine the values of internal forces. Therefore, such tasks must be solved applying the iteration method.
3. The calculation algorithm has been applied for one-span frame with a truss designed selecting standard profiles. The numerical experiment, under limits to the displacements of the characteristic nodes of the frame structure and under the evaluation of the stability of compressed bars, has been conducted.
4. The problem of analyzing the stressed-deformed state of the optimal frame structure has been resolved obtaining real displacements and internal forces. The boundary values of displacements have been established.
5. Aplastic deformation trajectory of the structure has been determined.

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TAMPRIOS PLASTINĖS RĖMINĖS KONSTRUKCIJOS OPTIMIZAVIMAS KOMPLEKSINĖMIS APRIBOJIMŲ SĄLYGOMIS

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Santrauka. Straipsnyje pateikiamas plieninių rėminių konstrukcijų strypų skerspjūvių optimizavimo uždavinio matematinis modelis, kuris leidžia įvertinti konstrukcijos stiprumo, standumo ir stabilumo reikalavimus. Optimizavimo uždavinio matematiniam modeliui sudaryti taikoma matematinio programavimo teorija ir ekstreminiai energiniai principai. Netiesiniam matematiniam programavimo uždaviniui spręsti taikoma *MatLab* programavimo aplinka. Dėl ryšio tarp konstrukcijos tampraus atsako dydžių ir optimizuojamų parametrų uždavinys sprendžiamas iteracijų būdu. Skaičiavimo algoritmas pritaikytas plokščiajam vieno tarpatramio rėmui su santvara. Rėminė konstrukcija diskretizuojama strypiniais baigtiniais elementais. Nustatytas minimalus konstrukcijos tūris, kai konstrukcija dar nepasiekusi visiško plastinio suirimo, tačiau atskiri jos elementai jau yra patyrę plastines deformacijas. Pagal gautąjį optimalų planą – strypų ribines įrąžas – parenkami standartiniai dėžiniai profiliuočiai.

Reikšminiai žodžiai: optimizacija, tamprioji plastinė rėminė konstrukcija, energiniai principai, stiprumas, standumas, stabilumas, *MatLab*.

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