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THE CORRELATION ANALYSIS OF THE ANGLE MEASUREMENTS

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Abstract. The test bench for testing and calibrating the measuring geodetic instruments for flat angle measurements has been created at Vilnius Gediminas Technical University, Institute of Geodesy. This test rig incorporates multiple-angular position determination principles – photoelectric angular encoder, polygon–autocollimator and circular-scale-microscope. Such a diversity of implemented methods allows cross-calibration and testing each principle using the other one. Similarly, there may be multiple methods of testing and cross-calibration used. The covariation and correlation analysis of the measurement data are received and analysed.

Keywords: circular scale, photoelectric encoder, polygon, autocollimator, measurement, evaluation.

1. Introduction

Precise geodetic instruments are widely used in geodesy, building structures, surveying, machine engineering etc. Many of these instruments, such as theodolites, tacheometers, total stations etc are based on precision angle measurements. For the purpose of accuracy assurance, these instruments are to be calibrated at appropriate periods of service time [1].

Test of these instruments is regulated by written standards [2, 3]. According to the standards, accuracy of the angle measurement performed by the instrument must be tested in the field conditions using the known length reference measure (triangulation principle). Using such method it is possible to get only a very restricted number of angular measurements; it does not allow to collect a large number of different (desired) angular values though geodetic instruments produce a vast number of discrete values on their display unit during measurement, and these values must be also checked [4, 5].

The calibration of geodetic measuring instruments requires a large number of angular values to be compared with the reference values. Only few test benches are used which allow to perform a complex testing and calibration of planar angle for geodetic instruments [6]. Such test rigs should have the angle determination (or positioning) precision much (3–5 times) higher than precision of tested (calibrated) devices.

Such test rig for testing and calibration of horizontal angle measurements of geodetic instruments has been created at Vilnius Gediminas Technical University, Institute of Geodesy. Since the precision (and accuracy) of the test rig during an angle determination must be very high, the determination of the accuracy parameters of the test rig itself is one of the most important tasks. In this paper we present some statistical correlation and covariation analysis of some test results.

2. Equipment for measurements

The test rig constructed operates as usual comparator device – it compares the angular position of the rotary table (6, Fig 1) with the readouts of geodetic instrument (4) mounted on the table. An angular positioning of rotary table of the test rig is performed by the worm-gear drive with the step motor (7). Angular position of the rotary table is determined by photoelectric angle encoder (8). The mode of operation of the test rig is controlled by PC (3) and a special control unit (9) controlling an operation of the step motor by pulses from the angular encoder. The test rig operates on an automated mode of operation.

Since installed angle encoder in most cases does not fulfil the requirements of the precision (and accuracy) needed for the test rig, several other angular position determination devices have been permanently installed. The measurements performed by 12-angle polygon (2) and the photoelectric autocollimator (1) are considered as the reference ones. An angular position of the table additionally can be determined by microscope (5) and the circular scale cut on the surface of the rotary table (6).

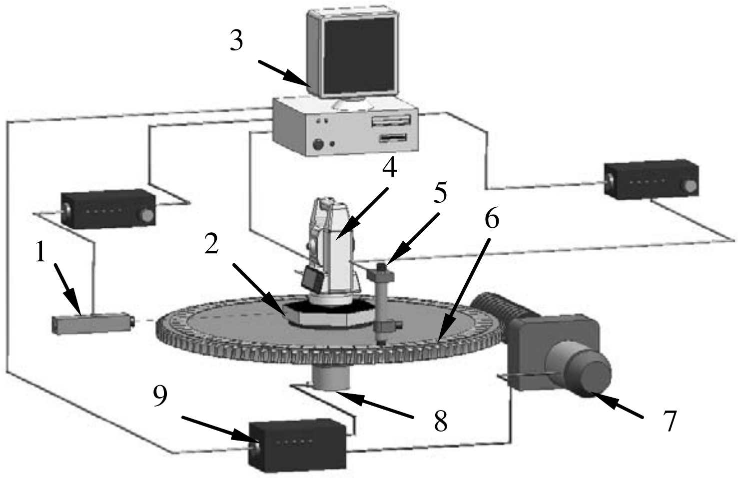

Fig 1. General diagram of the test rig

Normally rotary disk angular positioning by the drive according to pulses from the photoelectric angular encoder is considered as “rough”, and the precise angular value is determined by autocollimator and microscope. That way, the test rig is tested and calibrated by means of the installed instruments in it [7, 8]. Usual procedure of testing consists of positioning (by the encoder) of rotary disc at a pitch of 30° (due to the number of faces of multiangular prism) in full circle and registering the autocollimator and microscope readings at every position. The results of such tests are analysed further in this article.

3. Measurement data evaluation

The results received by means of polygon/autocollimator and circular scale/microscope measurements are analysed here. Four groups, each consisting of 3–4 of measuring tests were performed. The results were processed by means of statistical methods, calculating the matrixes of covariation and correlation coefficients for corresponding vectors. The circular scale tests were performed every 30°, ie 0°, 30°, 60°, …, 360°. The data matrix consists of autocollimator and microscope readings in each series. Columns of matrixes consist of the corresponding autocollimator and microscope measurement data. Rows of matrix from the autocollimator and microscope measurement data are obtained by changing rotary disc angular position at a pitch of 30°. The autocollimator and microscope measurements were obtained by measuring the position deviations from the corresponding nominal value. Therefore columns of data matrixes consist of the values of angle position measurements by autocollimator and microscope. The rows of data matrixes are considered as the corresponding measurement results of the values of the circular scale strokes position.

The arrays of covariation coefficients for corresponding measurement results, which are selected to the rows and columns, have been calculated from four test series. Evaluations of standard deviations of readings from autocollimator and microscope are shown in Table 1. Evaluations of standard deviations of readings from the circular scale at every 30° of angle are also shown in Table 1.

Evaluations of the covariation array of an appropriate values K_X are calculated according to formula:

$$K_X = \frac{1}{n-1} \delta X^T \delta X, \tag{1}$$

here δX – two-dimensional volume of data variation, which is calculated in such a manner:

$$\delta X = X - \frac{1}{n} X, \tag{2}$$

here X – two-dimensional volume of evaluates.

Evaluations of mutual covariation arrays of two variables X, Y are derived in this way:

$$K_{XY} = \frac{1}{n-1} \delta X^T \delta Y. \tag{3}$$

Generally form of covariation and correlation coefficient arrays for each series of autocollimator, microscope and scale strokes nominal values is:

$$K_{AM} = \begin{pmatrix} K_{11} & K_{12} & \dots & K_{1k} \\ K_{21} & K_{22} & \dots & K_{2k} \\ \dots & & \dots & \dots \\ K_{k1} & K_{k2} & \dots & K_{kk} \end{pmatrix}, \tag{4}$$

here K_{AM} – the covariation matrix of autocollimator and microscope readings.

The evalues of covariation coefficients have been calculated according to the programme written in *Matlab5* software. The calculations results are in Tables 2, 3.

The data provided in Tables 1–3 show that autocollimator standard deviations in all test series are on average twice smaller than the microscope ones. The data presented in Table 1 also show that evaluates of standard deviations of the readings at the circular scale's stroke in the position 0° are considerably (2–7 times) larger than the readings in the other scale strokes (30°, 60°, …, 330°) in all test series. It indicates the imprecise measurements in the initial stroke.

Table 1. Values of standard deviation (autocollimator and microscope readouts)

Scale stroke	Standard deviation (arc sec)
0°	12,1
30°	5,9
60°	5,5
90°	7,2
120°	3,9
150°	6,2
180°	6,8
210°	6,0
240°	7,7
270°	6,4
300°	6,3
330°	5,6
360°	11,8

Evaluations of the correlation coefficients presented in Tables 2–3 reveal that autocollimator and microscope measurements in all series are correlated, the values of coefficients of correlation being in the range of 0,5–0,99. The correlation coefficient values of approx 20 % of measurements are in the range of 0,05–0,30, therefore these data might be considered as uncorrelated, since the threshold value of standard deviation of correlation coefficient is $m_r \approx \left(1-r^2\right)/\sqrt{n} > 0,3$. These results might indicate that the measuring method is not entirely correct or the measurements were performed with errors caused by the operator.

The covariation between each scale stroke is presented in Fig 2. The correlation coefficient values of measurements performed at the scale stroke of 0° regarding all other measurements are negative. This is evident in all measurement series. Therefore the errors of measurements at initial 0° stroke have a negative influence on the errors at the other strokes of the circular scale. These data might also indicate some faults in the measurement method.

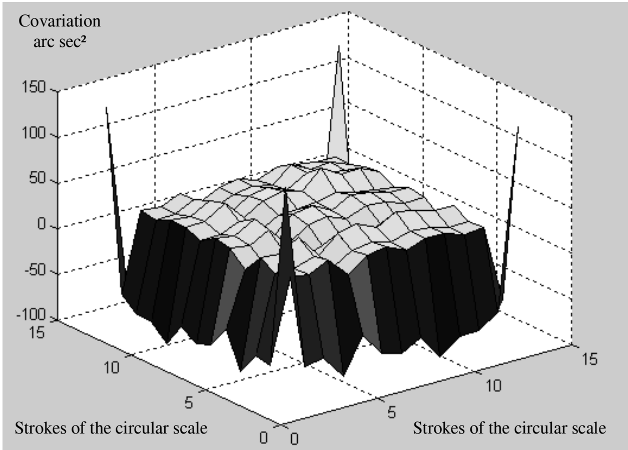

Fig 2. Diagram of evaluation of covariation matrix of the circular scale's strokes measurements

Table 2. Values of the correlation coefficients of autocollimator and microscope (using 4 series of tests)

1	0,55	0,88	0,51	0,96	0,58	0,88	0,53
0,54	1	0,34	0,97	0,53	0,99	0,33	0,98
0,87	0,34	1	0,41	0,90	0,41	0,90	0,35
0,51	0,97	0,41	1	0,53	0,98	0,35	0,97
0,95	0,53	0,90	0,53	1	0,60	0,82	0,50
0,58	0,99	0,41	0,98	0,60	1	0,36	0,96
0,87	0,33	0,90	0,35	0,82	0,36	1	0,40
0,52	0,98	0,35	0,97	0,50	0,96	0,40	1

Table 3. Correlation coefficients according to test rig rotary table position (calculated every 30°)

1	–0,11	–0,38	–0,60	–0,25	–0,62	–0,87	–0,35	–0,84	–0,78	–0,82	–0,093	0,99
–0,11	1	0,94	0,77	0,61	0,31	0,46	0,33	0,56	0,60	0,21	0,74	–0,11
–0,38	0,94	1	0,87	0,60	0,53	0,68	0,47	0,75	0,76	0,44	0,80	–0,39
–0,60	0,77	0,87	1	0,69	0,35	0,74	0,18	0,88	0,83	0,70	0,55	–0,60
–0,25	0,61	0,60	0,69	1	0,0027	0,56	–0,057	0,63	0,69	0,62	0,16	–0,18
–0,62	0,31	0,53	0,35	0,0027	1	0,71	0,95	0,61	0,63	0,40	0,61	–0,64
–0,87	0,46	0,68	0,74	0,56	0,71	1	0,52	0,96	0,97	0,86	0,35	–0,83
–0,35	0,32	0,47	0,18	–0,057	0,95	0,52	1	0,42	0,47	0,19	0,66	–0,37
–0,84	0,56	0,75	0,88	0,63	0,61	0,96	0,42	1	0,98	0,89	0,42	–0,82
–0,78	0,60	0,76	0,83	0,69	0,63	0,97	0,47	0,981	1	0,85	0,40	–0,74
–0,82	0,21	0,44	0,70	0,62	0,40	0,86	0,19	0,89	0,85	1	0,12	–0,79
–0,093	0,74	0,80	0,56	0,16	0,61	0,35	0,66	0,42	0,40	0,12	1	–0,16
0,99	–0,11	–0,39	–0,60	–0,18	–0,64	–0,83	–0,37	–0,82	–0,74	–0,79	–0,16	1

4. Conclusions

- Multiple angle measuring principles (photoelectric angle encoder, polygon/autocollimator and circular scale/microscope) have been implemented in the test rig allowing cross–calibration and testing of each other;
- Covariation and correlation analysis has shown that generally measurements taken by autocollimator and microscope have a good correlation;
- Some data (20 %) of autocollimator and microscope measurements have quite poor correlations (less than 0,30) showing imperfections in measuring process or methods;
- Poor correlations are present in the position (scale stroke) of 0° which negatively effects all other measurements (which are performed regarding 0°). Such shortcomings should be corrected in future measurements.

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