

## HOUSE AGE AND HOUSING PRICES: A VIEWPOINT OF THE OPTIMAL TIME FOR LAND REDEVELOPMENT

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**Abstract.** The value of a property comprises the value of both the building and the land. Numerous studies have reported a nonlinear relationship between house age and housing prices, which may result from mispricing the value of the land. The paper establishes a theoretical model to evaluate the optimal time for land redevelopment and the land value after redevelopment according to the real options. Although the depreciation effect causes the value of buildings to decrease as house age increases, properties with a lower residual building value have a higher probability of being redeveloped. Thus, the depreciation effect of building value and the inverse depreciation effect of land value contribute to the nonlinear relationship between house age and housing prices. Data from Taipei City, Taiwan, are employed for empirical analysis. The results imply that where land is scarce, housing prices are unlikely to depreciate due to ageing buildings.

**Keywords:** house age, depreciation effect, inverse depreciation effect, land redevelopment, real option.

### Introduction

The continued rise in housing prices is a common problem faced by many countries. This paper attempts to discuss why real estate does not depreciate and lose value like other capital goods, making real estate popular among the people. The value of real estate comprises the value of both the building and the land, so correctly estimating the value of the land is essential to the appraisal of real estate and the changing value of the real estate over time (i.e., the depreciation effect). However, land value is not easy to measure, especially in some housing markets in which land is scarce. This may be the reason why the depreciation effects found in the empirical literature are diverse. Some studies have confirmed the nonlinear effect of house age on housing prices (Goodman & Thibodeau, 1995; Coulson & McMillen, 2008).

Puzzles still remain regarding the nonlinear effect. Specifically, questions remain regarding the type of nonlinear effect, the causes of the nonlinear effect in different regions, and the reason behind the effect of house age on housing prices. This paper infers that this non-linear effect may be caused by a misappraisal of the value of land redevelopment. The goal of the present study is to establish a theoretical model to evaluate the optimal time for land redevelopment and the land value after redevelop-

ment according to the real options and then to rigorously re-examine the nonlinear effect of house age on housing prices to clarify the aforementioned three questions.

The value of a property consists of the values of the building and the land. In the literatures on housing prices, the value of land is relatively seldom discussed. However, the analysis of land value is very important (Muchová et al., 2018; Tu et al., 2021; Remeikienė et al., 2019), and changes in land price will affect other macroeconomic variables (Mirakatouli & Samadi, 2017; Zhang et al., 2020). Studies have focused on the depreciation effect, which describes the loss of property value through the aging of and damage to buildings as house age increases (Malpezzi et al., 1987). The nonlinear nature of depreciation has been discussed on the grounds that building maintenance costs mitigate building aging and damage. Wilhelmsson (2008) estimates the depreciation rates of housing prices among residential buildings according to their maintenance level and location. Francke and van de Minne (2017) contend that maintenance profoundly affects the physical damage sustained by residential buildings; for example, the value of a 50-year-old building without maintenance is reduced by 43%, but almost no physical damage is identified in a properly maintained house in the long term. Shilling et al. (1991) investigate the nonlinear differences among

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residential buildings in terms of their depreciation rates according to the maintenance motivations of their owners and renters; they determine that the early depreciation rate of a rented residential building is lower than that of an owned residential building.

None of the aforementioned studies have comprehensively clarified the nonlinear effect of house age on housing prices; in particular, the rise in housing prices as house age increases has not been explained. The depreciation effect only describes the decrease in housing price as house age increases, which constitutes a negative coefficient of influence. Appropriate maintenance technology and high maintenance costs only explain the nonlinear nature of the negative coefficient; they do not explain the appearance of a positive coefficient, which constitutes the inverse depreciation effect. However, some empirical studies have revealed that the increase in house age may lead to a rise in housing prices (e.g., Rehm et al., 2006), confirming the existence of the inverse depreciation effect. Because of the inconsistency among theoretical and empirical studies in their analyses of the effect of house age on housing prices, the inverse depreciation effect remains to be clarified.

Studies have employed the vintage effect to explain the puzzle of increasing house age possibly leading to a rise in housing prices. According to Rolheiser et al. (2020), a residential building's year of construction influences the period-specific collection of its characteristics, raw materials, and quality. Because of the relationship between house age, raw materials, and construction costs, some vintage residential buildings cannot be duplicated easily, limiting the supply of these buildings. Studies have indicated that residential buildings built in a specific period are particularly preferred by people, an effect similar to that found in the market for aged wine. According to Rolheiser et al. (2020), residential buildings constructed before 1900 or between 1900 and 1945 in the Netherlands are particularly sought-after. Studies on the vintage effect have explained the pricing phenomenon of houses constructed during specific periods; however, they have not provided an explanation for the effect of house age on housing prices that is consistent, systematic, and applicable to most houses.

Because no consistent explanation for the effect of house age on housing prices—a critical topic—has been developed, recent studies have focused on land value, which is a component of property value (e.g., Lee et al., 2005; Lai et al., 2021; Tong et al., 2021). And there are more and more recent documents discussing the issue of land redevelopment (e.g., Lu et al., 2020a; Zhong & Hui, 2021; Davis et al., 2021).<sup>1</sup> In the

future when land becomes scarce and scarce, if the price of land redevelopment is underestimated, it will cause greater bias in the estimation of housing prices.

Hence, in the present study, we aim to establish a theoretical model to evaluate the optimal time for land redevelopment for a property and its value after redevelopment according to the real options. Through the separate derivation of the changes in building values and land redevelopment values as house age increases, we deduce that the depreciation effect of building value and the inverse depreciation effect of land value are the causes of the nonlinear relationship between the age of properties and their values. A numerical analysis is conducted to clarify the time the inverse depreciation effect surfaces under different conditions. Actual data from Taipei City, the largest metropolis in Taiwan, are employed in the analysis, which reveals that the nonlinear effect of housing prices decreases first and then increases as house age increases; that is, the inverse depreciation effect occurs in old houses.

## 1. Literature review

### 1.1. Depreciation rate for housing

The effect describing the decline in asset price due to aging is called the depreciation effect (Hulten & Wykoff, 1981). Although the depreciation effect is a widespread phenomenon, this effect has very diverse impacts on real estate assets. In addition, because many theoretical models related to real estate require the depreciation rate for housing to be used as a variable, examining the depreciation effect is crucial when discussing the housing market. Many studies have examined the depreciation rate for housing (Margolis, 1982; Malpezzi et al., 1987). An increasing number of studies are revealing the complexity of the process of estimating the depreciation effect.

Through a theoretical and empirical examination of the relationship between house age and the market values of owned residential buildings, Goodman and Thibodeau (1995) report that the depreciation effect of residential buildings is nonlinear and that heterogeneity exists between them in terms of said effect. In the housing market, incorrect analysis of the depreciation effect leads to a hypothetical error in the use of the depreciation rate for housing as a variable and subsequent analysis bias. Randolph (1988) indicates that the depreciated costs of residential properties lead to erroneous consumer price index estimation. Knight and Sirmans (1996) report that overlooking the maintenance costs and price depreciation of different residential buildings leads to errors in housing price index estimation. Wilhelmsson (2008) stresses the importance of analyzing house depreciation rates, which can affect consumer and housing price index estimation; this estimation may in turn influence tax estimates and public policymaking.

Studies have explained the nonlinear phenomenon of the depreciation effect on the grounds that maintenance costs mitigate the aging of and damage to a building

<sup>1</sup> Lu et al. (2020a) use microdata on land acquisitions and developments of an emerging district in Taipei to discuss developer characteristics. Zhong and Hui (2021) propose a real option pricing model to improve the valuation of vertical mixed-use housing projects. Davis et al. (2021) use fairly complete data from 2012–2019 in the United States to calculate land prices and house the land share of the house value. Although these studies analyze land value or development, they do not primarily measure depreciation effects.

(Wilhelmsson, 2008; Knight & Sirmans, 1996; Francke & van de Minne, 2017). However, these studies have not explained the inverse depreciation effect, where housing prices increase as house age increases (Rehm et al., 2006; Lee et al., 2005). We deduce that the value of a property must be calculated in consideration of both the building value and land value. Relevant studies are unable to explain the inverse depreciation effect because they have only explored the loss of property value caused by the aging of and damage to buildings incurred through increasing house age.

## 1.2. Land value and redevelopment

According to Lee et al. (2005), the aging of a residential building not only leads to its depreciation but also increases the possibility of reconstruction. Expectations for reconstruction and the announcement of reconstruction plans strongly heighten the housing price of a building. Using the hedonic price model, Lee et al. (2005) divide the effect of house age into depreciation and reconstruction effects. Moreover, according to 3,474 transaction records in Seoul, South Korea, in 2001, they argue that the depreciation effect of houses aged 15–19 years is higher than their reconstruction effect, and the two effects will eventually reverse their influential power, causing housing prices to rise.

The empirical results reported by Lee et al. (2005) verify the inverse depreciation effect in Seoul's housing market. However, the theoretical model involves the assumption that the exogenous development probability function is positively correlated with house age, which implies that property redevelopment values are positively correlated with house age to explain the reconstruction effect. The model is incapable of using the endogenous decision function to explain the effect of house age on development probability and property values. Clapp and Salavei (2010) apply redevelopment options to comprehensively explain the redevelopment values of properties. Another exogenous variable, namely land use intensity, is implemented to describe redevelopment option values; lower land use intensity is assumed to lead to easier redevelopment and higher redevelopment values. Subsequent empirical studies have employed the concept of redevelopment options of Clapp and Salavei (2010). For example, it is applied by Munneke and Womack (2020) to estimate the capitalized values of the redevelopment options of residential buildings in exploring the spatial dynamics of housing prices.

Lee et al. (2005) and Clapp and Salavei (2010) do not examine the values of redevelopment options according to the decision-making behaviors of land owners. Rather, they directly hypothesize that land development probability and values are exogenous and positively correlated with house age. This analysis does not elucidate the positive effect of house age on property values; in other words, the endogenous decision-making model is incapable of explaining the inverse depreciation effect.

Other works of literature, such as Lu et al. (2020b) and Zhong and Hui (2021), also discuss the issue of land prices or land development. Still, they do not study the

phenomenon of depreciation effects. In the present study, a theoretical model is established to evaluate the optimal land redevelopment time and the value after redevelopment according to the real options, thereby supplementing the shortcomings of the aforementioned studies.

## 2. Theoretical model

Appropriately accessing real estate appraisals is always a crucial and central subject of real estate research. International Valuation Standards Council (IVSC) proposed three principal valuation approaches, which are market approach, income approach, and cost approach. Pagourtzi et al. (2003) point out that valuation methods can divide into two groups, which are traditional and advanced. The first includes those methods of comparable, cost, income, profit and contractor's approaches, and the other mentions hedonic pricing, spatial analysis, fuzzy logic and ARIMA models etc. Sayce et al. (2006) mention that, in terms of real estate, the word value could illustrate three different but related concepts: price, market value, and worth.

The value mentioned in this model is the market value assessed by the Cost Approach (CA). The concept of the CA is that a property value is generally assumed by taking for basis the cost of building a roughly similar property, which further presumes that the value of a property is equivalent to the cost of land plus the total cost of construction, subtracting depreciation if applicable. Many studies in the past have used this method to estimate the market value of real estate, for example, Moore (2012), Guo et al. (2014), Zujo et al. (2014), Burada and Demetrescu (2018), and Djurakulovich and Kizi (2021). And the method used in this paper is in accordance with International Standards principles proposed by IVSC: where the market value of an asset must reflect its highest and best use. Camposinhos and Oliveira (2019) point out that no matter for a new or the continuation of an asset's existing use or some alternative use, the highest and best use must be according to a real estate's potential and financial viability.

The theoretical model established in this study divides property value into two parts, namely building value and land value, and the changes in the two parts as house age increases are separately explored. When the highest and best use could be redeveloped, the IVSC claimed that the real estate value should include the value of the redevelopment of previously developed land. Hence, land values are usually estimated by using real options.

Real options are irreversible investments under uncertainty. In addition, they usually involve huge amount of capital. Several examples include American-style financial derivatives, policy adoption (Pindyck, 2000), and mortgage prepayment (Chen et al., 2009). To solve a real option problem, not only the value of the option, but also the optimal stopping time at which the option has to be exercised are derived. See for example, Ting et al. (2013), and Ewald and Wang (2010). There is no doubt that redevelopment is an irreversible investment which involves huge costs and

benefits. Consequently, the method of real options might explain the causes of the inverse depreciation effect. In addition, in this paper holdouts of land owners is not considered. Samsura and van der Krabben (2012) and Samsura (2013) propose that game theories are more appropriate to deal with the decisions of the land development case needs to include the participation of multiple stakeholders. Therefore, the model of this paper can be regarded as an analysis of single-family houses or as a preliminary study of multifamily homes without the landlord's holdouts behavior. Other articles analyzing real estate value in terms of real option also do not discuss the landlord's holdouts behavior of multifamily homes (Lee et al., 2005; Clapp & Salavei, 2010; Geltner et al., 2014, and Munneke & Womack, 2020).

To introduce our theoretical model, we begin with the conventional real estate valuation theory, which indicates that house age negatively affects housing price. Housing price comprises the value of building and the one of land. By adopting the CA, when a building is newly constructed, it is no need to consider the depreciation, hence its value is equal to its construction cost. Over time, the value of the building equals the difference of subtracting depreciation from reconstruction cost (replacement cost), that is the depreciated reproduction cost. On the other hand, the land value is determined by the rental income associated with the land. Let  $C$  be the rent per period and  $r$  the discount rate; at each period, the present value of the land remains constant at  $\frac{C}{r}$  and does not change with house age.

To illustrate the relationship between the housing price and the building value, we denote  $HP(t)$  and  $B(t)$  the housing price and the building value at time  $t$ , respectively. The initial time is  $t = 0$ , which implies that  $HP_0 = HP(0)$  and  $B_0 = B(0)$  represent the initial housing price and the initial building value, respectively. As mentioned earlier, the housing price consists of the following two values: the building value and the land value. In addition, the present value of the land is  $\frac{C}{r}$ . Therefore, in the conventional

real estate valuation theory, the relationship between the initial housing price and the initial land value is given by  $B_0 = HP_0 - \frac{C}{r}$ .

The ratio of the initial building value to housing price is expressed as  $\frac{B_0}{HP_0}$ . Let  $T$  be the service life of the building. We assume that the depreciation is apportioned through linear calculation, and the depreciation per year is expressed as  $\frac{B_0}{T}$ . The value of the building at time  $t$  can be expressed as  $B(t) = B_0 \left(1 - \frac{t}{T}\right)^+$ , and its housing price as:

$$HP(t) = B(t) + \frac{C}{r}. \tag{1}$$

Accordingly, house age negatively affects housing prices. Furthermore, the housing price decreases over time

and remains at the value  $\frac{C}{r}$  if  $t \geq T$ , which violates the empirical evidence of the inverse depreciation effect on housing price.

The above model is established by using the conventional real estate valuation theory, which fails to take any uncertainty and the inverse depreciation effect into consideration. One primary reason is the assumption of a constant land value. Benefits of land redevelopment, i.e., demolishing the existing building and constructing a new one, is random and a crucial factor which should be taken into account in models. Therefore, a model for calculating the optimal time point for land redevelopment is constructed to accurately estimate the land value. To land development companies, land redevelopment is an investment option, also known as a real option. Nevertheless, if the land value remains constant as the conventional real estate valuation theory suggests, the real option is always either deep in-the-money or deep out-of-the-money, which is not realistic. As a result, the assumption of a constant land rent must be replaced by the one of a stochastic land value. Let  $x(t)$  be the land value at time  $t$ . To simplify the analysis, the land value before and after redevelopment are supposed to follow two geometric Brownian motions:

$$\frac{dx(t)}{x(t)} = \mu_b dt + \sigma_b dW(t) \tag{2}$$

and

$$\frac{dx(t)}{x(t)} = \mu_a dt + \sigma_a dW(t). \tag{3}$$

According to Eqs (2) and (3), the uncertainty factor for the land value can be expressed using the continuous variable  $W(t)$ , which follows a Wiener process. The parameters  $\mu_b$  and  $\mu_a$  are interpreted as the average growth rates of the land value before and after development, respectively;  $\sigma_b$  and  $\sigma_a$  represent the volatility of the land value before and after development, respectively. We assume that  $\mu_a > \mu_b$  and  $\sigma_a > \sigma_b$ , which indicate that the average growth rate and volatility of land value increase after development. The following assumptions are formulated in regard to the behaviors of land development companies:

**A1.** Land developers are risk-neutral parties, and the final development time is  $\hat{T} > T$ .

**A2.** With an existing building, the land exhibits the input cost of  $I(t)$  at  $t$ , which includes the cost of demolishing the building and the residual value of the loss of the building. In other words,  $I(t) = K_0 e^{\delta t} + B_0 \left(1 - \frac{t}{T}\right)^+$ , where  $K_0 > 0$  represents the combined labor and equipment cost of land redevelopment when  $t = 0$ , and  $\delta \geq 0$  represents the annual growth of the cost.

**A3.** The benefit the developer gains from the land value at  $t$ , such as land rental, is the fixed ratio of the land value at  $t$ , expressed as  $\alpha x(t)$ , where  $\alpha \in (0,1)$ .

According to the aforementioned assumptions, the value of the land development option  $V(t, x)$  is calculated. In A1,  $\hat{T}$  can be infinite. Before the land is developed,

the accumulated benefit expected by the developer is expressed as follows:

$$V(t, x) = \max_{\tau \in [t, T]} E \left\{ \int_t^\tau e^{-r(\tau-s)} \alpha x(s) ds + e^{-r(\tau-t)} (\bar{V}(\tau, x(\tau)) - I(\tau)) | x(t) = x \right\}, \quad (4)$$

where:  $E\{\cdot|\cdot\}$  represents the conditional expectation;  $\tau$  represents the time of land redevelopment;  $\bar{V}(t, x)$  represents the accumulated benefit expected by the developer after the land development at  $t$  and land value  $x$ . To simplify the analysis, the following hypotheses are formulated:

A4. After the land is redeveloped at  $\tau$ , the redevelopment stops within  $[\tau, \hat{T}]$ .

A5. The annual discount rate  $r$  exceeds  $\mu_a$  and  $\mu_b$ .

According to A4, the calculation process of  $\bar{V}(t, x)$  can be simplified as follows:

$$\bar{V}(t, x) = E \left\{ \int_t^{\hat{T}} e^{-r(\tau-s)} \alpha x(s) ds | x(t) = x \right\}. \quad (5)$$

A5 prevents the divergence of the discounted benefit accumulated in Eq. (5) when  $\hat{T}$  is infinite. The solution to Eq. (5) can be acquired through the following partial differential equation:

$$r\bar{V}(t, x) - \bar{V}_t(t, x) = \alpha x + \mu_a x \bar{V}_x(t, x) + \frac{\sigma_a^2 x^2}{2} \bar{V}_{xx}(t, x),$$

$$\bar{V}(\hat{T}, x) = 0. \quad (6)$$

This leads to  $\bar{V}(t, x) = \frac{\alpha}{r - \mu_a} \left[ 1 - e^{-(r - \mu_a)(\hat{T} - t)} \right] x$ ,

which can be applied in Eq. (4) to obtain the following partial differential equations:

$$rV(t, x) - V_t(t, x) = \alpha x + \mu_b x V_x(t, x) + \frac{\sigma_b^2 x^2}{2} V_{xx}(t, x); \quad (7a)$$

$$V(t^*, x^*) = \frac{\alpha}{r - \mu_a} \left[ 1 - e^{-(r - \mu_a)(\hat{T} - t^*)} \right] x^* - I(t^*)$$

(value-matching condition); (7b)

$$V_x(t^*, x^*) = \frac{\alpha}{r - \mu_a} \left[ 1 - e^{-(r - \mu_a)(\hat{T} - t^*)} \right]$$

(smooth-pasting condition), (7c)

where  $(t^*, x^*)$  indicates the optimal time for land redevelopment; that is, when the time reaches  $t^*$  and the land value attains  $x^*$ , the land should be redeveloped. The aforementioned partial differential equations can be solved using numerical methods, such as the finite difference method.

The housing price that includes the land redevelopment value is expressed as follows:

$$HP(t, x) = B(t) + V(t, x), \quad (8)$$

where  $B(t)$  decreases following an increase in the house age  $t$  and  $V(t, x)$  increases following an increase in the land value. Generally, the land value rises following an increase in  $t$ . Therefore,  $V(t, x)$  generally increases over time. As time passes, when the effect of land value increase on the housing price is greater than the negative effect of house age on said price, inverse depreciation occurs. Eq. (8) explains the nonlinear relationship between housing price  $HP(t, x)$  and house age  $t$  as well as the potential cause of the inverse depreciation effect.

### 3. Numerical analysis

This paper discusses the nonlinear relationship between housing price  $HP(t, x)$  and house age  $t$  as well as the inverse depreciation effect through the endogenization of land development decision making. This discussion also enables comparative analyses under different contexts to describe the factors influencing the extent of the inverse depreciation effect. Therefore, the theoretical model described in the previous section is applied to analyze the changes in average housing prices over time with different discount rates ( $r$ ) and different building value to housing price ratios ( $\frac{B_0}{HP_0}$ ). The effects of several parameters on the

average housing prices and the derivation are presented in Appendix. We employ the Monte Carlo method and Eq. (8) to simulate the land value per year and its corresponding housing price 1,000,000 times. When calculating the average housing price, we exclude the situation where no housing price is present because the existing building has been demolished during land redevelopment. Finally, because the real option is calculated through the finite difference method, if the simulated land value per year does not fall into the grid of this method, it should be calculated through linear interpolation. The values of parameters used in the Monte Carlo simulation can be referred to in Table 1.

Table 1. The parameters used in the Monte Carlo method

The expected growth rate of the land rent appreciation-before redevelopment ( $\mu_b$ )	The expected growth rate of the land rent appreciation-after redevelopment ( $\mu_a$ )	The volatility of the land rent appreciation-before redevelopment ( $\sigma_b$ )	The volatility of the land rent appreciation-after redevelopment ( $\sigma_a$ )	The ratio of the land rent ( $\alpha$ )
1%	2%	10%	20%	5%
The life span of the building ( $T$ )	The initial value of the building ( $B_0$ )	The initial labor cost of redevelopment ( $K_0$ )	The growth rate of the labor cost ( $\delta$ )	The maturity of the redevelopment option ( $T$ )
30	1	0.2	1%	250

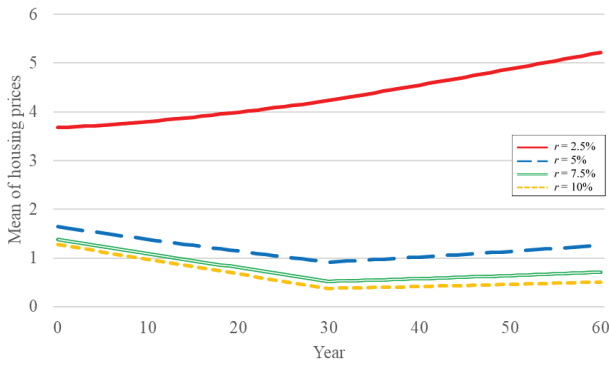


Figure 1. The effect of house age on the average housing price at different discount rates

Figure 1 illustrates the effect of house age on the average housing price at different discount rates, namely 2.5%, 5%, 7.5%, and 10%. The initial land value is  $x(0) = 0.5$ . As depicted in the figure, the mentioned effect is nonlinear, and after the service life of the building (30 years in this study) expires, the housing price increases over time. Additionally, higher discount rates lead to lower average price lines (Figure 1). This is possibly because the higher the discount rate, the lower the discounted value of the expected cumulative land rent. Because no significant relationship exists between the land redevelopment cost and the discount rate, the land value must increase for the developer to be willing to redevelop the land. Therefore, the land development option value decreases when the discount rate increases. Because the housing price is the sum of the building and land redevelopment option values, the average housing price decreases when the discount rate increases. The average housing prices with the discount rates of 5%, 7.5%, and 10% decrease first before increasing when the house age (service life) increases, forming the inverse depreciation effect. On the other hand, the average housing price with the discount rate of 2.5% only increases as the house age increases, and the nonlinear effect is not noticeable. This is because the low discount rate raises the value of the land redevelopment option. Therefore, the initial decrease in building value does not affect the housing price significantly. The analysis results depicted in Figure 1 indicate that the inverse depreciation effect is influenced by the discount rate; that is, low discount rates may lead to the inverse depreciation effect in houses of low age.

Figure 2 depicts the effect of house age on the average housing price with different initial building values (measured according to  $\frac{B_0}{HP_0}$ ). Possible reasons, such as a shorter life span of the building, a lower growth rate of the labor cost and a higher expected growth rate of the land value, lead to a lower  $\frac{B_0}{HP_0}$ . Therefore, studying the effect of  $\frac{B_0}{HP_0}$  on average housing price allows us to take into different parameters into account. With the initial building value fixed at  $B_0 = 1$ , we calculate different  $\frac{B_0}{HP_0}$  ra-

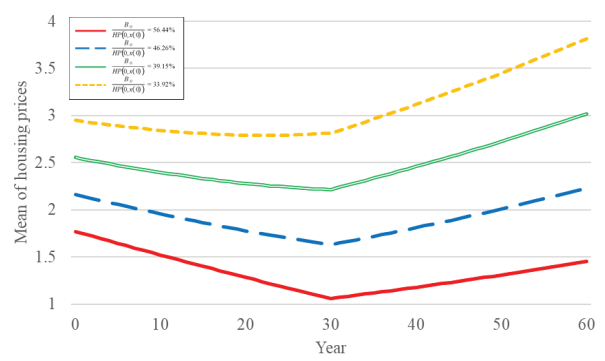


Figure 2. The effect of house age on the average housing price with different initial building values

tios using different initial land values, namely 1, 1.5, 2, and 2.5 with the discount rate of  $r = 7.5\%$ . Thus, the corresponding  $\frac{B_0}{HP_0}$  ratios are 56.44%, 46.26%, 39.15%, and 33.92%, respectively. A higher  $\frac{B_0}{HP_0}$  indicates a higher initial building value. As demonstrated in the figure, regardless of the  $\frac{B_0}{HP_0}$  ratio, the inverse depreciation effect exists in the average housing price. A lower initial building value leads to a higher average price line. With a fixed initial building value, a lower  $\frac{B_0}{HP_0}$  ratio indicates that higher housing prices lead to higher land redevelopment option values. Therefore, a lower initial building value leads to a higher average price line as well as to a more noticeable inverse depreciation effect. Accordingly, the inverse depreciation effect is influenced by  $\frac{B_0}{HP_0}$ ; that is, a lower  $\frac{B_0}{HP_0}$  indicates a stronger inverse depreciation effect.

#### 4. Empirical results

This study employs data from Taipei City, the largest metropolis in Taiwan, to verify the inverse depreciation effect. Similar to Singapore and Hong Kong, Taiwan is characterized by its small habitable spaces and high population density. In these regions of Asia, habitable land is precious, which has led to high housing prices. Therefore, discussions on land redevelopment are urgent and of paramount importance in Taiwan, which is an emerging market in Asia. Taipei City, Taiwan’s most important administrative, economic, and financial center, has the highest housing prices in the country. However, because Taipei is one of the earliest-developed cities in Taiwan, over 50% residential buildings have exceeded 30 years of age as of 2020. With a high average house age, the residential buildings in Taipei City continue to rise in price, and the existence of the depreciation effect in the city’s housing market has been called into question. Therefore, this study employs all 18,441 items of transaction data in Taipei City from 2019 to 2020 to analyze the inverse depreciation effect in housing prices.

Table 2. Descriptive statistics

	<i>HP</i>	<i>Age</i>	<i>Size</i>	<i>Floor</i>
Mean	2354.9580	25.2931	38.3800	5.6554
Std. Dev.	2612.6000	16.0539	26.6699	4.0844
Skewness	6.2606	-0.0913	3.2132	1.5591
Kurtosis	71.9242	1.8092	28.6146	6.6863
	<i>Hall</i>	<i>Room</i>	<i>BRoom</i>	
Mean	1.6237	2.5092	1.5912	
Std. Dev.	0.6759	1.2594	0.8981	
Skewness	-0.3044	0.5511	6.4714	
Kurtosis	7.6152	7.0954	217.4929	

Notes: *HP*, *Age*, *Size*, *Floor*, *Hall*, *Room*, and *BRoom* denote housing price, house age, house size, residential floors, number of halls, number of rooms, and number of bathrooms, respectively.

A massive amount of comprehensive data is employed to ensure robust analysis results. The data encompass the 12 administrative districts of Taipei City. The variables in these data must be measured and controlled to accurately identify the effect of house age on housing prices. Therefore, this paper, like other empirical literature, also uses the housing price characteristic model to estimate (e.g., Aziz et al., 2021; Malaitham et al., 2020; Liang & Yuan, 2021). In addition to house age, past articles studying housing prices in Taipei found that house size (Lin & Hwang, 2004; Lin, 2004),<sup>2</sup> the number of living rooms (Hong & Lin, 1999; Wu et al., 2017), number of rooms (Hong & Lin, 1999; Lee et al., 2017), and number of bathrooms (Hong & Lin, 1999) would affect the house prices.

Therefore, in this paper, the hedonic housing price model features the variables of house age, house size, residential floors, number of halls, number of rooms, and number of bathrooms as well as control variables for different districts. Table 2 lists the simple statistics of all the variables excluding the district-specific variables. The average house age for all the samples is 25.29 years; the average house size is 38.38 Taiwanese ping, which is 126.884 m<sup>2</sup>; and the average housing price is NT\$ 23.55 million. These values indicate the high price, small size, and old age of Taipei's residential buildings.

Table 3 lists the preliminary least squares regression analysis results on the effects of all the explanatory variables on housing prices. Except linear (Edmonds Jr., 1984) model, there are still other forms of models which can estimate the least squares regression, for example, Log-Linear (Nelson, 1978), Log-Log (Case & Quigley, 1991),

<sup>2</sup> To consider the influence of residential floors on house prices, some studies use the number of floors (Lin, 2004), some papers use dummy variables to examine the effect of the first floor (Hong & Lin, 1999) or the fourth floor (Lee et al., 2017). Because the first floor might have a commercial value and, in Mandarin, the pronunciation of "four" floor is similar to "death". After estimating different variables, this paper selects the variable, the number of floors, which lets the model perform better.

and Box-Cox (Cassel & Mendelsohn, 1985) models. Economic theory does not provide a suitable functional form for hedonic pricing functions, according to Rosen (1974), Freeman (1979), Halvorsen and Pollakowski (1981), and Cassel and Mendelsohn (1985). As a result, we can experiment with different functional forms before settling on the optimum multiple regression equation. Malpezzi (2008) purposed the reasons for the popularity of the log-linear functional. Hence, we also use the log-linear function to estimate for comparison, and the results are listed in Appendix Table A1. The results obtained by log-linear model show that the fit of the model is poor, because the adjusted  $R^2$  is only 0.7187. On the other hand, the adjusted  $R^2$  of the linear model is 0.77. In addition to the high level of model fitness, some past studies also used the linear model when describing the capitalization impact of specific variables on house prices, which is able to more intuitively explain how significantly the factor affects the price, such as Olden and Tamayo (2014), Higgins et al. (2019), and Zou (2019). Hence, this paper adopted the linear method to estimate for hedonic pricing.

Table 3 shows all the explanatory variables influence housing price significantly. Moreover, buildings with more floors, of a larger size, and greater number of bathrooms exhibit significantly higher housing prices. However, the numbers of halls and rooms negatively affect housing prices. This is possibly because residential buildings of the same size are less likely to be luxury properties when they have more halls and rooms, and these buildings are likely to be houses with low total prices inhabited by more people. Finally, house age positively affects housing prices with an estimated coefficient of 4.88, indicating that the housing price increases by NT\$ 48,800 per year. Accordingly, the housing market in Taipei City exhibits no depreciation effect, but it does exhibit a noticeable inverse depreciation effect.

In addition, the theoretical model of this paper infers that there should be a nonlinear effect existing in the depreciation effect of housing prices, that is, the house price decreases first and then increases as house age increases, causing the inverse depreciation effect occurs in old houses.

Table 3. OLS regression

Variables	Coefficient	Std. Error	t-Statistic	p-value
Age	<b>4.8793</b>	0.7096	6.8763	0.0000
Size	<b>85.6525</b>	0.4178	205.0266	0.0000
Floor	<b>40.2252</b>	2.5512	15.7675	0.0000
Hall	<b>-165.9736</b>	17.2209	-9.6379	0.0000
Room	<b>-259.7113</b>	11.9115	-21.8034	0.0000
BRoom	<b>105.8648</b>	14.4358	7.3335	0.0000
District <sub>1</sub>	<b>-525.4219</b>	46.9061	-11.2016	0.0000
District <sub>2</sub>	<b>-582.8951</b>	53.8018	-10.8341	0.0000
District <sub>3</sub>	<b>507.5995</b>	49.6463	10.2243	0.0000
District <sub>4</sub>	<b>-272.4294</b>	42.6515	-6.3873	0.0000
District <sub>5</sub>	<b>-199.1980</b>	58.6763	-3.3949	0.0007
District <sub>6</sub>	<b>-743.9364</b>	41.2750	-18.0239	0.0000
District <sub>7</sub>	<b>-1130.5390</b>	46.8398	-24.1363	0.0000
District <sub>8</sub>	<b>-1024.7780</b>	44.1699	-23.2008	0.0000
District <sub>9</sub>	<b>-157.0037</b>	54.3491	-2.8888	0.0039
District <sub>10</sub>	28.9902	53.0537	0.5464	0.5848
District <sub>11</sub>	<b>-969.4232</b>	61.6671	-15.7203	0.0000
District <sub>12</sub>	<b>-951.9235</b>	49.8039	-19.1134	0.0000
adj. R <sup>2</sup>	0.7720			

Notes: Age, Size, Floor, Hall, Room, and BRoom denote house age, house size, residential floors, number of halls, number of rooms, and number of bathrooms, respectively. District<sub>1-12</sub> are control variables for different districts, which denote Shilin, Datong, Daan, Zhongshan, Zhongzheng, Neihu, Wenshan, Beitou, Songshan, Xinyi, Nangang, and Wanhua Districts, respectively. Numbers in bold denotes statistically significant at 5% level.

Table 4. Multiple threshold tests

Sequential F-statistic determined thresholds: 4			
Threshold test	F-statistic	Scaled F-statistic	Critical value
0 vs. 1	<b>122.5090</b>	<b>2205.1610</b>	27.03
1 vs. 2	<b>34.5051</b>	<b>621.0924</b>	29.24
2 vs. 3	29.9091	<b>538.3628</b>	30.45
3 vs. 4	2.1815	<b>39.2676</b>	31.45
4 vs. 5	0.0000	0.0000	32.12

Notes: Numbers in bold denotes statistically significant at 5% level.

In order to more rigorously analyze the nonlinear phenomenon of housing prices, this paper also used the quantile model to estimate, and the estimated results are listed in Appendix Figure A1. Koenker and Bassett (1978) as well as Koenker and Hallock (2001) proposed a method to examine the nonlinearity effect among different levels of a dependent variable, which are used to estimate how the dependent variable is affected by the independent variable under different quantiles. Liao and Wang (2012) also used the method to find the heterogeneity in the housing prices. As we can see in Appendix Figure A1, it is true that house prices under different components are affected differently by explanatory variables. From the perspective of house age, the higher the house price, the less negatively affected by the age of the house. This may be because the higher the housing price, the more likely it is to be located on land with high development value, and the less affected by depreciation.

Then, in order to objectively test the inference of this paper, we first test whether the nonlinear threshold effect exists, and then endogenously estimate the house ages that the depreciation phenomenon changes. Therefore, the existence of the nonlinear effect in the regression model illustrated in Table 3 is verified, as demonstrated in Table 4, through conducting multiple threshold tests. These tests evaluate whether house age can be used as a threshold variable in measuring the effects of the explanatory variables on housing prices. The sequential F-statistic results indicate that the effects of the explanatory variables on housing prices are significant and nonlinear, and house age can be applied as a threshold variable to distinguish these nonlinear effects. The optimal model fit test results reveal a total of four notable threshold values. In other words, the relationship between housing price and the other variables changes as house age increases; when properties exceed various thresholds in age, they start to exhibit different housing price characteristics.



Table 5. Threshold regression

Variables	Age < 6.6 (N = 3,382)		6.6 ≤ Age < 22.8 (N = 4,497)		22.8 ≤ Age < 36.3 (N = 4,352)		36.3 ≤ Age < 42.3 (N = 3,443)		42.3 ≤ Age (N = 2,767)	
	Coefficient	t-Stat.	Coefficient	t-Stat.	Coefficient	t-Stat.	Coefficient	t-Stat.	Coefficient	t-Stat.
Age	<b>-45.1780</b>	-3.8642	1.6376	0.4102	<b>-15.6704</b>	-3.4411	<b>25.7220</b>	2.2031	<b>47.6946</b>	9.4391
Size	<b>90.8107</b>	149.3051	<b>77.5367</b>	84.8493	<b>58.9670</b>	41.8776	<b>65.9259</b>	53.5002	<b>92.4931</b>	53.8272
Floor	<b>84.4131</b>	20.5648	<b>22.2411</b>	5.0336	<b>16.7863</b>	3.3134	12.4198	1.5791	<b>-70.9064</b>	-5.4531
Hall	-65.4199	-1.3861	-62.2509	-1.4337	0.1370	0.0038	-14.3455	-0.4190	<b>-116.8482</b>	-3.6177
Room	<b>-423.2713</b>	-14.5281	<b>-232.0740</b>	-8.9787	-26.1193	-0.9954	<b>-70.1282</b>	-2.6629	<b>-145.7669</b>	-6.0730
BRoom	<b>187.0664</b>	8.0542	<b>209.8163</b>	5.0084	-4.0753	-0.1203	6.8206	0.2091	<b>78.4592</b>	2.6002
District <sub>1</sub>	<b>-711.0068</b>	-8.2269	<b>-597.7462</b>	-5.6419	313.6769	1.9506	<b>-1494.4620</b>	-3.1047	<b>-2435.6780</b>	-8.9179
District <sub>2</sub>	<b>-789.9922</b>	-9.2787	<b>-615.1284</b>	-5.5647	107.3050	0.5946	<b>-1521.6900</b>	-3.1267	<b>-2654.5270</b>	-9.2237
District <sub>3</sub>	<b>1759.1670</b>	17.3651	<b>512.6054</b>	5.4928	<b>1088.6720</b>	6.5346	-438.2692	-0.9222	<b>-2067.6070</b>	-7.0948
District <sub>4</sub>	-78.1956	-0.9918	<b>-224.4731</b>	-2.6389	<b>475.0414</b>	2.9736	<b>-1199.5790</b>	-2.5288	<b>-2291.1390</b>	-8.3391
District <sub>5</sub>	<b>-596.9550</b>	-4.6412	36.1150	0.3384	<b>658.8042</b>	3.7141	-870.2380	-1.8004	<b>-2396.4240</b>	-8.1610
District <sub>6</sub>	<b>-1192.5840</b>	-17.7301	<b>-779.6940</b>	-8.5761	91.2626	0.6133	<b>-1400.0440</b>	-2.9816	<b>-2080.7760</b>	-6.8771
District <sub>7</sub>	<b>-1807.1910</b>	-14.3599	<b>-1215.3900</b>	-13.3765	-187.5010	-1.2982	<b>-1637.6720</b>	-3.4901	<b>-2786.2670</b>	-9.5077
District <sub>8</sub>	<b>-1659.1080</b>	-20.8541	<b>-1140.3850</b>	-11.5149	-193.8865	-1.2444	<b>-1768.9040</b>	-3.6913	<b>-2708.6370</b>	-9.8673
District <sub>9</sub>	<b>4621.7290</b>	12.4252	-115.8071	-1.0484	<b>845.5506</b>	5.0433	<b>-961.2042</b>	-1.9969	<b>-2293.3790</b>	-8.1969
District <sub>10</sub>	<b>-664.6621</b>	-3.6793	<b>1072.4360</b>	10.0425	<b>636.6800</b>	4.0432	<b>-1083.3200</b>	-2.2481	<b>-2295.7640</b>	-8.2532
District <sub>11</sub>	<b>-1624.8050</b>	-10.0737	<b>-906.3707</b>	-9.6225	137.4019	0.7196	<b>-1486.4290</b>	-2.8991	<b>-2470.4930</b>	-8.0835
District <sub>12</sub>	<b>-1592.9660</b>	-18.1183	<b>-1173.8960</b>	-10.6266	-33.3772	-0.1908	<b>-1514.2000</b>	-3.1626	<b>-2646.6630</b>	-9.8414

Notes: Age, Size, Floor, Hall, Room, and BRoom denote house age, house size, residential floors, number of halls, number of rooms, and number of bathrooms, respectively. District<sub>1-12</sub> are control variables for different districts. Numbers in bold denotes statistically significant at 5% level.

Table 5 depicts the threshold regression model estimation results, which present the properties distinguished by the aforementioned four thresholds with the highest fit (Table 4). The estimation results provide a comprehensive explanation of the nonlinear effect of house age on housing prices. Among the samples, the 3,382 houses younger than 6.58 years are negatively affected by house age in their housing prices by an estimated coefficient of -45.18. Accordingly, the depreciation effect exists in the prices of houses younger than 6.58 years, and the price depreciates by NT\$ 450,000 per year. However, the depreciation effect diminishes as the house age increases. When the house age exceeds 6.58 years but remains lower than 36.27 years, housing price depreciation either becomes nonsignificant or reduces to a rate of NT\$ 150,000 per year. The most critical threshold for house age is 36.27 years, beyond which the depreciation effect completely vanishes and the inverse depreciation effect begins. The housing prices of houses aged between 36.27 and 42.33 years rise by NT\$ 250,000 per year. For houses older than 42.33 years, the estimated coefficient becomes 47.69, indicating that housing prices increase by nearly NT\$ 480,000 per year.

For further clarification, the estimated coefficients on the effect of house age on housing prices under the four threshold values are illustrated in Figure 3, where the horizontal axis represents house age and the vertical axis represents the estimated coefficient (i.e., the depreciation

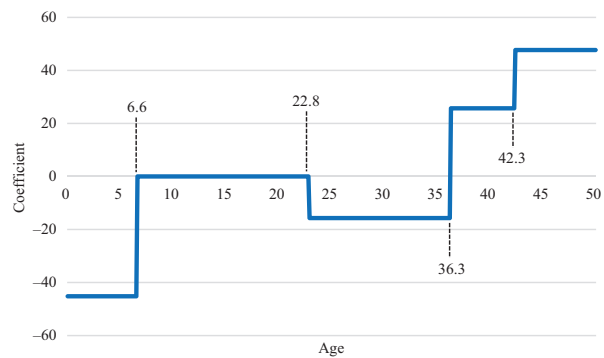


Figure 3. The depreciation amount per year

amount per year listed in Table 5). In Figure 3, the nonlinear effect of house age on housing prices is clearly illustrated. Only the depreciation effect exists in houses no older than 36.27 years; for those older than 36.27 years, housing prices increase every year, which constitutes an inverse depreciation effect. The empirical results listed in Figure 3 are consistent with the simulation results listed in Figures 1 and 2. Moreover, the estimation results listed in Figure 3 reveal that the properties in Taipei City are aged approximately 36 years, and the values of the buildings have depreciated to nearly zero, facilitating land redevelopment value.

Table 6. OLS regression (using dummy variable for age)

Variables	Coefficient	Std. Error	t-Statistic	p-value
<i>DAge</i> <sub>1</sub>	<b>-241.6675</b>	29.7639	-8.1195	0.0000
<i>DAge</i> <sub>2</sub>	<b>-228.3984</b>	31.7885	-7.1849	0.0000
<i>DAge</i> <sub>3</sub>	<b>-147.9337</b>	34.0451	-4.3452	0.0000
<i>DAge</i> <sub>4</sub>	<b>257.2902</b>	37.3527	6.8881	0.0000
<i>Size</i>	<b>84.5606</b>	0.4185	202.0492	0.0000
<i>Floor</i>	<b>42.8132</b>	2.5389	16.8630	0.0000
<i>Hall</i>	<b>-145.7410</b>	17.1429	-8.5016	0.0000
<i>Room</i>	<b>-253.8980</b>	11.7972	-21.5218	0.0000
<i>BRoom</i>	<b>121.0395</b>	14.3658	8.4255	0.0000
<i>District</i> <sub>1</sub>	<b>-357.8784</b>	47.6049	-7.5177	0.0000
<i>District</i> <sub>2</sub>	<b>-475.1334</b>	54.2109	-8.7645	0.0000
<i>District</i> <sub>3</sub>	<b>711.9856</b>	50.6612	14.0539	0.0000
<i>District</i> <sub>4</sub>	<b>-95.0671</b>	44.1177	-2.1549	0.0312
<i>District</i> <sub>5</sub>	-16.3687	59.7334	-0.2740	0.7841
<i>District</i> <sub>6</sub>	<b>-538.7442</b>	42.8180	-12.5822	0.0000
<i>District</i> <sub>7</sub>	<b>-893.2570</b>	49.5019	-18.0449	0.0000
<i>District</i> <sub>8</sub>	<b>-877.6786</b>	44.7975	-19.5921	0.0000
<i>District</i> <sub>9</sub>	40.3676	55.3833	0.7289	0.4661
<i>District</i> <sub>10</sub>	<b>233.8356</b>	54.5384	4.2875	0.0000
<i>District</i> <sub>11</sub>	<b>-760.0944</b>	64.7111	-11.7460	0.0000
<i>District</i> <sub>12</sub>	<b>-843.8939</b>	50.3139	-16.7726	0.0000
<i>adj. R</i> <sup>2</sup>	0.7754			

Notes: *DAge*<sub>1</sub> is a dummy variable, which equals 1 if the house age is between 6.6 and 22.8, 0 otherwise. *DAge*<sub>2</sub> is a dummy variable, which equals 1 if the house age is between 22.8 and 36.3, 0 otherwise. *DAge*<sub>3</sub> is a dummy variable, which equals 1 if the house age is between 36.3 and 42.3, 0 otherwise. *DAge*<sub>4</sub> is a dummy variable, which equals 1 if the house age is above 42.3, 0 otherwise. *Size*, *Floor*, *Hall*, *Room*, and *BRoom* denote house size, residential floors, number of halls, number of rooms, and number of bathrooms, respectively. *District*<sub>1–12</sub> are control variables for different districts, which denote Shilin, Datong, Daan, Zhongshan, Zhongzheng, Neihu, Wenshan, Beitou, Songshan, Xinyi, Nangang, and Wanhua Districts, respectively. Numbers in bold denotes statistically significant at 5% level.

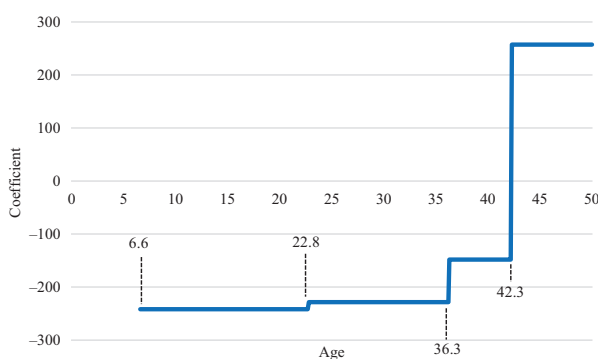


Figure 4. The depreciation amount per year (OLS model with dummy variables)

In addition, this paper adopted another way to estimate the nonlinear influence of the house age on housing prices, as suggested by previous studies (e.g., Trojane et al., 2018). Table 6 shows the estimated results by examining the threshold effects using dummy variables in one equation. For further clarification, the estimated coefficients on the effect of house age on housing prices under

the different threshold values of house age are drawn in Figure 4. Table 6 and Figure 4 indicate the following: It is evident that as the house ages are higher, the negative depreciation effect of housing prices is more negligible. And in the estimation result of the last threshold value (the house age is above 42.3), there is a positive effect of house age on housing prices.

### Conclusions

This study examines the effect of house age on housing prices through theoretical and empirical analyses. The results reveal the nonlinear effect of house age on housing prices as well as the circumstances in which depreciation and inverse depreciation effects occur. We take advantage of real option theory to construct a theoretical model to analyze the optimal time for land redevelopment and the value of it. The housing price takes into account the value of building and that of redevelopment. According to the model, as the house age increases, the building value initially decreases because of a significant depreciation effect. However, the lower the building value, the closer the property might be to the optimal redevelopment time, and the

value of right to land for compensation starts to surface. Consequently, the increase in house age leads to a greater opportunity to develop the land, thus causing the inverse depreciation effect to become more apparent.

The depreciation effect of building value and the inverse depreciation effect of land value reveal a nonlinear relationship between the age and value of properties. The 18,441 items of comprehensive transaction data in Taipei City from 2019 to 2020 are employed for empirical verification, yielding robust analysis results. The results confirm the depreciation effect of new houses and the inverse depreciation effect of old houses. The threshold values for the change of effects and the intensity of the effects are also revealed. The empirical results demonstrate that as the house gets older, the effect of decreasing house prices (the depreciation effect) exists, but when the house is old enough, the effect of rising house prices (the inverse depreciation effect) also exists. The depreciation effect is the most substantial in houses aged 6 years or younger, and the inverse depreciation effect is the most considerable in houses aged 36 years or older.

Numerous puzzles continue to exist regarding the effect of house age on housing price. For example, the causes of the nonlinear effect, factors in the differences among regions in terms of the nonlinear effect, and forms and characteristics of the nonlinear effect of house age on housing prices require clarification. The present study can address the aforementioned puzzles through a reexamination of the effect of house age on housing prices through theoretical and empirical analysis.

In this paper, we find that the increase in house age affects the two types of value differently, causing a nonlinear relationship between house age and housing prices. Because the proportions of new and old houses vary in each region, regions that are developed later than others have higher proportions of new houses and have a more apparent depreciation effect. By contrast, Taipei City has a higher proportion of old houses, and thus, the inverse depreciation effect is particularly noticeable. As indicated by the numerical simulation analysis and empirical estimation in the model, the depreciation effect comes before the inverse depreciation effect in the nonlinear relationship between house age and housing prices. Finally, the paper illustrates that the factors influencing the characteristics of the nonlinear effect are discount rate and the ratio of building value to housing price. When the discount rate is low, the inverse depreciation effect becomes noticeable in new houses. On the other hand, the lower the ratio of building value to housing price, the greater the inverse depreciation effect becomes.

Accordingly, the results of this paper contribute to the literature understanding of the impact of age on housing prices. Future studies are suggested to examine market samples that are appropriate for investigating the depreciation effect and the inverse depreciation effect effects of house age. Besides, the results of this paper imply that if land is scarce, housing prices are unlikely to depreciate

due to ageing buildings. Therefore, in these areas where land is scarce, it is difficult to reduce the demand for real estate, and of course, the price is not easy to fall.

There are lots of factors influencing housing prices, particularly qualitative factors which are represented by using dummy variables. Nevertheless, the problems of collinearity are more likely caused if the number of dummy variables is huge. This implies that an empirical model cannot be estimated accurately. Therefore, several factors, such as housing types, are not taken into consideration in our empirical model. On the other hand, some data, unfortunately, are unavailable, which is a limitation of this study. For instance, quality of apartment of information on the renovation of the buildings. Therefore, future research may focus on case studies to obtain more detailed analyses.

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### Author contributions

WW contributed formal analysis, investigation, and writing. IT contributed data curation, analysis, and writing.

### Conflicts of interest

The authors declare no conflict of interest.

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## Appendix

In this appendix, we present the effects of several parameters in Table 1 on the housing prices, i.e., Eq. (8). Following Wang (2020), a trinomial recombining tree which satisfies the local consistency conditions can be established to approach Eqs (2) and (3) in probability. Let  $\varepsilon > 0$  be the length of each period in the tree model and  $t_i = i\varepsilon$ , where  $i = 0, 1, 2, \dots$ . If  $x_{i,j}$  denotes the  $j^{\text{th}}$  node at time  $t_i$ , where  $j = 1, 2, \dots, 2i + 1$ , then the relationship among nodes at  $t_i$  and  $t_{i+1}$  is

$$x_{i+1,k} = \begin{cases} ux_{i,j}, & \text{if } k = j \\ x_{i,j}, & \text{if } k = j+1, \\ dx_{i,j}, & \text{if } k = j+2 \end{cases} \quad (\text{A.1})$$

where  $u = e^{a\sigma_b\sqrt{\varepsilon}} = \frac{1}{d}$  and  $a > 1$  is the dispersion parameter. Regarding the transition probability of  $x_{i,j}$  moving to  $x_{i+1,k}$ , it is defined as:

$$p_{i,j}^{i+1,k}(\mu_l, \sigma_l) = \begin{cases} \frac{\sigma_l^2 + (1-d)u_l}{(u-1)(u-d)}\varepsilon, & \text{if } k = j \\ 1 - \frac{\sigma_l^2 + (1-d)u_l}{(u-1)(u-d)}\varepsilon - \frac{\sigma_l^2 - (u-1)u_l}{(1-d)(u-d)}\varepsilon, & \text{if } k = j+1, \\ \frac{\sigma_l^2 + (u-1)u_l}{(1-d)(u-d)}\varepsilon, & \text{if } k = j+2 \end{cases} \quad (\text{A.2})$$

where  $l = a, b$ . To simplify the analysis, we assume that  $a, u_l$  and  $\sigma_l$  are selected so that  $p_{i,j}^{i+1,k}(u_l, \sigma_l) \in [0, 1]$ . If they are given such that  $p_{i,j}^{i+1,k}(u_l, \sigma_l) \notin [0, 1]$ , Propositions 2.1–2.3 in Wang (2020) can be applied to re-define the transition probabilities. The details are omitted here. It is highlighted that the case of  $p_{i,j}^{i+1,k}(u_l, \sigma_l) \in [0, 1]$  occurs when  $|\mu_l|$  is sufficiently large. This implies that the land value changes monotonically and significantly, which is less likely in practice.

Let  $t_N$  and  $V_{i,j}$  denote the terminal time and the approximation of Eq. (A.4) at node  $x_{i,j}$ , respectively.  $V_{i,j}$  is evaluated by

$$V_{i,j} = \max \left\{ \bar{V}(t_i, x_{i,j}) - I(t_i), \alpha x_{i,j} \varepsilon + e^{-r\Delta t} E \left\{ V_{i+1,k} | x_{i,j} \right\} \right\}, \quad (\text{A.3})$$

and

$$V_{N,j} = \max \left\{ \bar{V}(t_N, x_{N,j}) - I(t_N), 0 \right\}. \quad (\text{A.4})$$

In Eq. (A.3), the term  $\bar{V}(t_i, x_{i,j}) - I(t_i)$  is called the exercise value, and  $\alpha x_{i,j} \varepsilon + e^{-r\Delta t} E \left\{ V_{i+1,k} | x_{i,j} \right\}$  is the continuation value. One advantage of the tree model follows: The trinomial tree models approaching Eqs (A.3) and (A.4) share the same nodes. The only difference is the transition probabilities. Even though the transition probabilities, i.e., Eq. (A.2), depend on  $u_l$  and  $\sigma_b$ , the exercise value in Eq. (A.3) is dependent of the nodes, instead of the transition probabilities. In addition, the derivation of continuation value in Eq. (A.3) require the transition probabilities  $p_{i,j}^{i+1,k}(\mu_b, \sigma_b)$ , instead of  $p_{i,j}^{i+1,k}(\mu_a, \sigma_a)$ . As a result, it becomes more mathematically tractable when solving Eq. (A.3).

We now move on to finding the effects of parameters on the housing prices. We begin with the following proposition:

*Proposition 1. If  $\hat{T}$  is given sufficiently large, then the housing price  $HP(t, x)$  is decreasing in the discount rate  $r$ .*

*Proof.* It is noted that each node  $x_{i,j}$  and the transition probabilities in the tree model are independent of  $r$ . On the other hand, the exercise value  $\bar{V}(t, x) - I(t)$  is decreasing in  $r$ , given a sufficiently large  $\hat{T}$ . This can be verified by differentiating  $\bar{V}(t, x) - I(t)$  with respect to  $r$ . Therefore, it results from Eq. (A.4) that a higher  $r$  leads to a lower  $V_{N,j}$ . It then follows Eq. (A.3) that  $V_{i,j}$  is decreasing over  $r$ . Finally, since  $V_{i,j}$  approximates  $V(t_i, x_{i,j})$  and  $B(t)$  is independent of  $r$ , the housing price  $HP(t, x) = B(t) + V(t, x)$  declines over the discount rate  $r$ .

The interpretation of Proposition 1 follows: A higher discount rate leads to a lower value of the redevelopment option, which therefore reduces the housing price.

We now study the effects of the ratio of the land rent ( $\alpha$ ) and the expected growth rates of the land rent ( $\mu_a, \mu_b$ ) on the housing price. We present the following proposition:

*Proposition 2. The housing price is increasing in  $\alpha, \mu_a$  and  $\mu_b$ .*

*Proof.* We begin with the case of  $\alpha$ . Following Eq. (A.4), the terminal value rises in  $\alpha$ . Therefore, the exercise value

in Eq. (A.3) is larger given a higher  $\alpha$ . This results in a higher  $V_{i,j}$ . The case of  $\mu_a$  is analogous to the one of  $\alpha$  and we omit here. Regarding the case of  $\mu_b$ , it can be seen that a higher  $\mu_b$  leads to a higher  $p_{i,j}^{i+1,j}(\mu_b, \sigma_b)$  as well as a lower  $p_{i,j}^{i+1,j+1}(\mu_b, \sigma_b)$  and a lower  $p_{i,j}^{i+1,j+2}(\mu_b, \sigma_b)$ . This raises the exercise value in Eq. (A.3), since the expectation, i.e.,  $E \left\{ V_{i+1,k} | x_{i,j} \right\}$  increases. To conclude,  $V_{i,j}$  is increasing in  $\mu_b$ . Finally, since the housing price is  $HP(t, x) = B(t) + V(t, x)$  and  $B(t)$  is independent of  $\alpha, \mu_a$  and  $\mu_b$ ,  $HP(t, x)$  is increasing in the three parameters.

It is noted that a larger  $\alpha$  leads to a bigger land rent. In addition, since the land rent is proportional to the land value, higher expected growth rates of the land value, i.e.,  $\mu_a$  and  $\mu_b$ , also lead to a higher land rent. As a result, the continuation value rises in  $\alpha, \mu_a$  and  $\mu_b$ , which then raises  $V_{i,j}$ . To conclude, Proposition 2 holds.

Regarding the parameters which influence the cost  $I(t)$ , i.e.,  $K_0, \delta, B_0$  and  $T$ , we present the following proposition:

*Proposition 3. The value of the redevelopment option is decreasing in  $K_0, \delta, B_0$ , and is increasing in  $T$ .*

*Proof.* It is noted that a large  $K_0, \delta, B_0$  imply a higher cost  $I(t)$ , which leads to a lower exercise value. On the other hand, if the redevelopment option is exercised at  $t_{i+1}$  given  $x_j$ , then the continuation value at  $t_i$  given  $x_j$  reduces. Therefore,  $V_{i,j}$  declines in  $K_0, \delta, B_0$ . On the other hand, a higher  $T$  raises  $\bar{V}(t, x)$  and reduces  $I(t)$ , which leads the exercise value to become larger. As a result,  $V_{i,j}$  is increasing in  $T$ . This completes the proof.

The interpretation of Proposition 3 follows: A higher initial labor cost, growth rate of labor cost and initial building value, i.e.,  $K_0, \delta, B_0$ , all raise the cost of redevelopment. This reduces the value of the redevelopment option. On the other hand, the shorter the life span of the building is, the sooner the building value depreciates. As the cost of redevelopment consists of demolishing the building, the cost of redevelopment reduces. Therefore, the value of the redevelopment option rises. Since the housing price is the sum of the values of the building and the redevelopment option, the housing price is decreasing in  $K_0$  and  $\delta$ . Nevertheless, it is highlighted that the effects of  $B_0$  and  $T$  on the housing price are ambiguous.

Finally, we study the effects of the volatilities of the land values after and before redevelopment, i.e.,  $\sigma_a$  and  $\sigma_b$ . In the former case, it follows from the assumption A1 that  $\sigma_a$  does not affect  $\bar{V}(t, x)$ , i.e., the expected accumulated benefit gained by the developer after the land development. This implies that the housing price is independent of  $\sigma_a$ . Regarding the latter case, we present the following proposition:

*Proposition 4. The housing price is increasing in  $\sigma_b$ .*

*Proof.* To prove the proposition, we apply the Taylor expansions of  $u = e^{a\sigma_b\sqrt{\varepsilon}} = \frac{1}{d}$ :

$$u = 1 + a\sigma_b\sqrt{\varepsilon} + \frac{a^2\sigma_b^2}{2}\varepsilon + O(\varepsilon^{1.5}), \quad (\text{A.5})$$

and

$$d = 1 - a\sigma_b \sqrt{\varepsilon} + \frac{a^2 \sigma_b^2}{2} \varepsilon + O(\varepsilon^{1.5}). \tag{A.6}$$

Substituting Eqs (A.5) and (A.6) into  $p_{i,j}^{i+1,k}(\mu_b, \sigma_b)$ , where  $k = j, j+1, j+2$ , lead to  $p_{i,j}^{i+1,j} = \frac{1}{2a^2} + O\sqrt{\varepsilon}$ ,  $p_{i,j}^{i+1,j+1} = 1 - \frac{1}{a^2} + O\sqrt{\varepsilon}$  and  $p_{i,j}^{i+1,j+2} = \frac{1}{2a^2} + O\sqrt{\varepsilon}$ . At  $t_{N-1}$ , since

$$p_{i,j}^{i+1,j} u + p_{i,j}^{i+1,j+1} + p_{i,j}^{i+1,j+2} d = \left(\frac{1}{2a^2} + O\sqrt{\varepsilon}\right) e^{a\sigma_b \sqrt{\varepsilon}} + \left(1 - \frac{1}{a^2} + O\sqrt{\varepsilon}\right) + \left(\frac{1}{2a^2} + O\sqrt{\varepsilon}\right) e^{-a\sigma_b \sqrt{\varepsilon}} = \frac{1}{2a^2} + \left(e^{a\sigma_b \sqrt{\varepsilon}} + e^{-a\sigma_b \sqrt{\varepsilon}}\right) + 1 - \frac{1}{a^2} + O\sqrt{\varepsilon},$$

we have  $E\{V_{N,k} | x_{N-1,j}\}$  increases in  $\sigma_b$ . This leads to a higher exercise value, which raises  $V_{N-1,j}$ . It then follows

mathematical induction that  $V_{i,j}$  is increasing in  $\sigma_b$ . Therefore, the housing price increases in  $\sigma_b$ .

Proposition 4 indicates that a higher volatility of the land value before redevelopment raises the housing price. A possible interpretation follows: A higher volatility results in a bigger change in land value. However, if the land value is lower due to a higher volatility, the developer is given the right to postpone exercising the redevelopment option. On the other hand, a higher volatility leads to a larger exercise value when the redevelopment option is exercised. Therefore, the redevelopment option is more valuable. Since the value of the building is independent of the volatility of land value, the housing price is increasing. We summarize the results of Propositions 1–4 in Table A2.

Table A1. OLS regression (Dependent variable:  $\ln HP$ )

Variables	Coefficient	Std. Error	t-Statistic	p-value
Age	<b>-0.0102</b>	0.0002	-44.3598	0.0000
Size	<b>0.0190</b>	0.0001	139.7727	0.0000
Floor	<b>0.0057</b>	0.0008	6.8320	0.0000
Hall	<b>0.1082</b>	0.0056	19.3673	0.0000
Room	<b>0.0522</b>	0.0039	13.5120	0.0000
BRoom	-0.0023	0.0047	-0.4970	0.6192
District <sub>1</sub>	<b>6.6766</b>	0.0152	438.5170	0.0000
District <sub>2</sub>	<b>6.5793</b>	0.0175	376.7951	0.0000
District <sub>3</sub>	<b>7.0186</b>	0.0161	435.5474	0.0000
District <sub>4</sub>	<b>6.7207</b>	0.0138	485.4516	0.0000
District <sub>5</sub>	<b>6.8511</b>	0.0190	359.7544	0.0000
District <sub>6</sub>	<b>6.6187</b>	0.0134	493.8523	0.0000
District <sub>7</sub>	<b>6.4210</b>	0.0152	422.3074	0.0000
District <sub>8</sub>	<b>6.4532</b>	0.0143	449.8004	0.0000
District <sub>9</sub>	<b>6.9016</b>	0.0176	391.2305	0.0000
District <sub>10</sub>	<b>6.8199</b>	0.0172	396.0493	0.0000
District <sub>11</sub>	<b>6.6075</b>	0.0200	330.1489	0.0000
District <sub>12</sub>	<b>6.3796</b>	0.0162	394.6626	0.0000
adj. R <sup>2</sup>	0.7185			

Notes: Age, Size, Floor, Hall, Room, and BRoom denote house age, house size, residential floors, number of halls, number of rooms, and number of bathrooms, respectively. District<sub>1–12</sub> are control variables for different districts, which denote Shilin, Datong, Daan, Zhongshan, Zhongzheng, Neihu, Wenshan, Beitou, Songshan, Xinyi, Nangang, and Wanhua Districts, respectively. Numbers in bold denotes statistically significant at 5% level.

Table A2. The effects of parameters on the building value, the redevelopment option and the housing price. “+” and “-” stand for a positive effect and a negative effect, respectively

Parameter	Effect on building value	Effect on redevelopment option	Effect on housing price
The discount rate ( $r$ )	No effect	-	-
The expected growth rate of the land rent appreciation before redevelopment ( $\mu_b$ )	No effect	+	+
The expected growth rate of the land rent appreciation after redevelopment ( $\mu_a$ )	No effect	+	+
The volatility of the land rent appreciation before redevelopment ( $\sigma_b$ )	No effect	+	+

End of Table A2

Parameter	Effect on building value	Effect on redevelopment option	Effect on housing price
The volatility of the land rent appreciation after redevelopment ( $\sigma_a$ )	No effect	No effect	No effect
The ratio of the land rent ( $\alpha$ )	No effect	+	+
The life span of the building ( $T$ )	-	+	Ambiguous
The initial value of the building ( $B_0$ )	+	-	Ambiguous
The initial labor cost of redevelopment ( $K_0$ )	No effect	-	-
The growth rate of the labor cost ( $\delta$ )	No effect	-	-



Figure A1. Quantile regression coefficients