

SIMPLIFIED TECHNIQUE FOR CONSTITUTIVE ANALYSIS OF SFRC

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Abstract. Steel fibre reinforced concrete (SFRC) has become widespread material in areas such as underground shotcrete structures and industrial floors. However, due to the absence of material models of SFRC reliable for numerical analysis, application fields of this material are still limited. Due to interaction of concrete with fibres, a cracked section is able to carry a significant portion of tensile stresses, called the residual stresses. In present practices, residual stresses used for strength, deflection and crack width analysis are quantified by means of standard tests. However, interpretation of these test results is based on approximation using empirically deduced relationships, adequacy of which might be insufficient for an advanced numerical analysis. Based on general principles of material mechanics, this paper proposes a methodology for determination of residual stress-crack opening relationships using experimental data of three-point bending tests. To verify the constitutive analysis results, a numerical modelling is utilised employing a nonlinear finite element analysis program ATENA. Simulated load-crack width relationships and moment-curvature diagrams were compared with the experimental data by validating adequacy of the derived constitutive models.

Keywords: steel fibre reinforced concrete, constitutive analysis, residual stresses, flexural members, crack width, deformations.

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Introduction

Steel fibre reinforced concrete (SFRC) is a cement based composite material reinforced with discrete randomly distributed fibres. A sufficient quantity of fibres (normally greater than 15 kg/m³) added to the conventional concrete may have both technical and economic advantages, such as improved ductility, corrosion resistance and substantial reduction of laborious reinforcement work (Gribniak *et al.* 2013b). Despite its extensive and long-term use in specific areas, e.g. industrial floors and underground shotcrete structures, steel fibres have not occupied the general market of concrete structures. One of the main reasons is that the major design codes do not cover SFRC structures.

Quantification of the post-cracking strength of SFRC (provided by interaction between fibres and concrete) becomes a rather complicated issue due to diversity in the shape, material properties and bond characteristics of fibres (Gribniak *et al.* 2012). Post-cracking strength as the main parameter, describing behaviour of SFRC, appears because of its capability to sustain tensile stresses due to interaction between concrete and fibres crossing the crack plane. Tensile stresses, which appear in a crack-

ed SFRC, are often referred as residual. For investigation of post cracking behaviour of SFRC, both direct and inverse techniques can be used. The direct analysis is dedicated to the prediction of SFRC response using the specified material models, whereas the inverse analysis aims at determination of the parameters of a material model employing the constitutive test results (Kaklauskas *et al.* 2011).

Generally, SFRC can be considered as a concrete with randomly dispersed fibres or as a homogeneous material – conventional concrete with improved material properties. In the first case, post-cracking response of the material is defined through meso-scale models (Lofgren 2005; Jones *et al.* 2008; Fuggini *et al.* 2013). At this research level, the bridging of the crack is based on the number of fibres across the crack plane and on concrete and fibre bond interaction properties during the crack opening. Involving a large number of generally stochastic parameters, such models tend to be very complex. Moreover, computational implementation of these models is time consuming. Therefore, the research carried out on the macro-scale and being associated to the inverse analysis is commonly proceeded analysing SFRC as a homogeneous material. Most of the models found in literature

propose simple and continuous non-differentiable constitutive diagrams that are characterised through macroscopic material properties. These approaches focus either on residual stress-strain ($\sigma_{fr}-\epsilon$) (RILEM 2003; Dupont 2003) or on residual stress-crack opening width ($\sigma_{fr}-w$) relationships (DBV 2001; RILEM 2002a; DafStb 2010).

Several test methods may be used to determine, directly or indirectly, residual strength of cracked SFRC. The axial tension test appears to be the most fundamental and straightforward method to define the fracture properties of a material, as the tensile stress-crack width ($\sigma_{fr}-w$) relationship can be directly determined from the test results. However, the axial tension test is rather complicated to perform and it has been shown that the experimental results are affected by the test equipment and specimen interaction (RILEM 2001; Dupont 2003; Pujadas *et al.* 2013). Typically, bending tests of standard size (small in respect of full scale elements) beam specimens are the most currently used to characterize the post-cracking response of SFRC. These can be based either on three-point bending tests on notched or un-notched specimens (RILEM 2003; CEN 2005) or on four-point bending tests (DBV 2001; CNR 2007). Based on these test results and the inverse analysis techniques, the tensile strength and the residual stresses are defined, which can be further used for the strength, deflection or crack width analysis of SFRC members (Dupont 2003; Slowik *et al.* 2006). It should be pointed out that a large scatter of the test results often accompanies the standard experiments on SFRC. It mainly depends on the number of fibres crossing the crack plane (Di Prisco *et al.* 2006; Gribniak *et al.* 2012).

Seeking to reduce the scatter of experimental results, an alternative and novel approach for obtaining residual stresses from beam tests was recently proposed by the authors (Gribniak *et al.* 2012, 2013c). Unlike the standard techniques based on tests of small bending specimens, the proposed approach uses data of moment-curvature diagrams obtained from flexural tests on full scale SFRC beams with bar reinforcement. Solving the inverse deformation problem, when the model parameters are selected in accordance to the given response of a real structure, an average stress-average strain relationship of SFRC in tension is derived. As it can be seen from the Figure 1, the obtained stresses for the SFRC members at a given strain consist of the stresses due to the tension-stiffening effect and the residual stresses due to fibre and concrete interaction. Thus, residual stresses can be quantified as the difference of stresses obtained for the SFRC and conventional RC elements. In this figure, f_{ct} and ϵ_{cr} are the reference tensile strength and cracking strain, respectively; σ_{fr} is the residual tensile stress due to fibre interaction with concrete; σ_{ct} and ϵ_{ct} are the average stress and average strain of concrete in tension.

It can be pointed out that the fibre effect on tension-stiffening is related with two aspects: type and content of fibres and structural composition of the section. With the increasing area of bar reinforcement, relative influence of fibres to the stiffness of cracked element will decrease (Gribniak *et al.* 2012; Lee *et al.* 2013). With decreasing

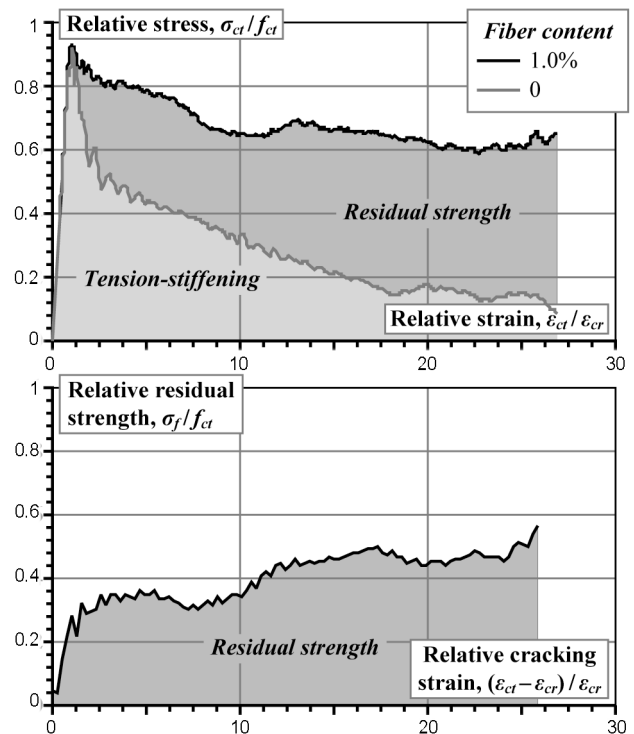


Fig. 1. Average stress-average strain response of tensile steel fibre reinforced concrete obtained from tests of beams with ordinary reinforcement bars (Gribniak *et al.* 2012)

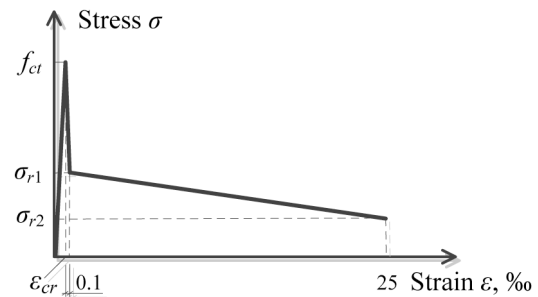


Fig. 2. Stress-strain relationship proposed in RILEM (2003)

cover of bar reinforcement, effectiveness of fibres might increase due to the restrained longitudinal crack opening. Application of micro-fibres can also give a positive effect on bond strength (Chao *et al.* 2009). In general, higher amount of fibres might increase the residual stresses; however, it can lead to higher possibility of structural defects (for instance, balling effect) and therefore reduce the bond strength. Holschemacher *et al.* (2010) and Ganesan *et al.* (2014) performed extensive investigation on this issue.

The current paper is dedicated to the determination of post-cracking behaviour of SFRC in tension. Seeking universality, the methodology for constitutive analysis of SFRC is proposed. The developed inverse technique determines residual stresses of SFRC in mechanically sound manner. Adequacy of the proposed technique was validated using experimental data of standard (small) and full-scale beams employing the derived residual strength models into nonlinear finite element analysis program ATENA as material law of SFRC in tension.

1. Constitutive modelling of SFRC

One of the ways to define the post-cracking behaviour of SFRC was proposed by Naaman (2003). A simple analytical expression for residual strength was suggested accounting pull-out length ratio, the efficiency factor of fibre orientation in respect to the crack plane, and the group reduction factor associated with the number of fibres pulling out per unit area of the crack:

$$f_r = \lambda_1 \cdot \lambda_2 \cdot \lambda_3 \cdot \tau \cdot F; \quad F = \frac{V_f \cdot l_f}{b_f \cdot \beta}, \quad (1)$$

where: λ_1 is the expected pull-out length ratio; λ_2 is the fibre orientation factor; λ_3 is the group reduction factor associated with the number of fibres pulling out per unit area; τ is the average bond stress of a single fibre embedded in the concrete; V_f is the fibre volume percentage; β is the bond factor, accordingly to Campione (2008), can be assumed being equal to 0.5 for round fibres, 0.75 – for crimped fibres, 1.0 – for hooked fibres; l_f and b_f are the length and the diameter of fibre, respectively. It is important to note that expression (1) gives constant value of the residual stresses being unacceptably rough for solving the advanced analysis problems.

Another method for assessing tensile behaviour of cracked SFRC was proposed in RILEM (2003). The method uses a tri-linear stress-strain relation of SFRC in tension and is derived from the experimental data of three-point bending test on the specimens with dimensions of 150×150×600 mm. To control the cracking process, a notch at mid-span of the beam is formed that allows measuring the opening crack width. The post-cracking response of SFRC is characterised by residual stresses σ_{r1} and σ_{r2} (Fig. 2), which correspond to experimental load values at different beam deformation stages at certain mid-span deflection (or crack width) values defined by the RILEM (2002b). The residual stresses are determined by following expressions:

$$\sigma_{r1} = 0.675 \cdot k \cdot \frac{P_1 \cdot L}{b \cdot h_{sp}^2}; \quad \sigma_{r2} = 0.555 \cdot k \cdot \frac{P_2 \cdot L}{b \cdot h_{sp}^2}, \quad (2)$$

where: k is the scale factor; P_1 and P_2 are, respectively, the loads recorded at the mid-span deflection equal to $\delta_1 = 0.46$ mm and $\delta_2 = 3.0$ mm (Fig. 3a) or crack width equal to $w_1 = 0.5$ mm and $w_2 = 3.5$ mm (Fig. 3b); L and b are the span and the width of specimen, respectively; h_{sp} is the depth of notched section.

2. Experimental program

An experimental program was conducted in order to investigate the effect of steel fibres on the response of members subjected to bending. Several beams were tested, containing 40 kg/m³ and 80 kg/m³ hooked-ended steel fibres (respectively equivalent to 0.5% and 1.0% of the total specimen volume). Fibres with a length l_f of 50 mm and a diameter d_f of 1 mm, resulting in the aspect (length to diameter) ratio of 50 were used.

Standard 150×150×600 mm beams with the span of 500 mm were casted according to RILEM (2002a). Displacement controlled testing machine was used and

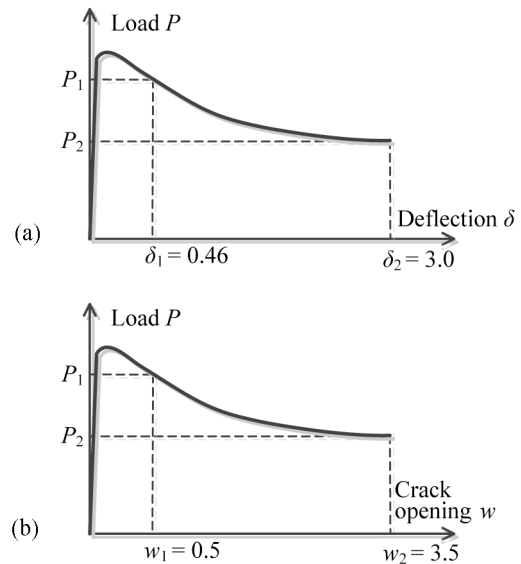


Fig. 3. Experimentally obtained load-deflection (a) and load-crack width (b) relationships (RILEM 2002b)

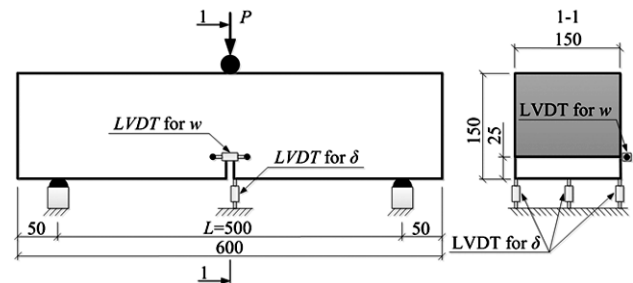


Fig. 4. The test set-up

beams were loaded with the rate of 0.2 mm/min. To localize cracking process, each beam was equipped with 25 mm notch at the mid-span. The mid-span deflections as well as crack opening width were measured within the test using linear variable displacement transducers (LVDTs). The test set-up is shown in Figure 4.

The compressive strength of concrete was measured at the age of 28 days on 150 mm cube specimens, resulting in average values of 43.1 MPa and 44.8 MPa for 0.5 and 1.0% steel fibre concrete mixes, respectively. From the test results of the individual beams in the test series, load-crack width curves have been constructed. Figures 5a and 5b show the load-crack width curves for 0.5 and 1.0% of fibre content, respectively. It can be observed that for most of the specimens the cracking resistance increases with increased volume of fibres. However, the variability in the obtained diagrams is significant. Such variety of results was mainly influenced by the orientation and number of fibres acting at the crack section.

3. Proposed inverse analysis technique

In the current study, an inverse procedure has been proposed to perform constitutive analysis of SFRC in tension. The technique employs experimental data from standard 3-point bending tests and uses simple equilibrium of axial forces and bending moments to find unknown values of the parameters defining the residual stresses in SFRC.

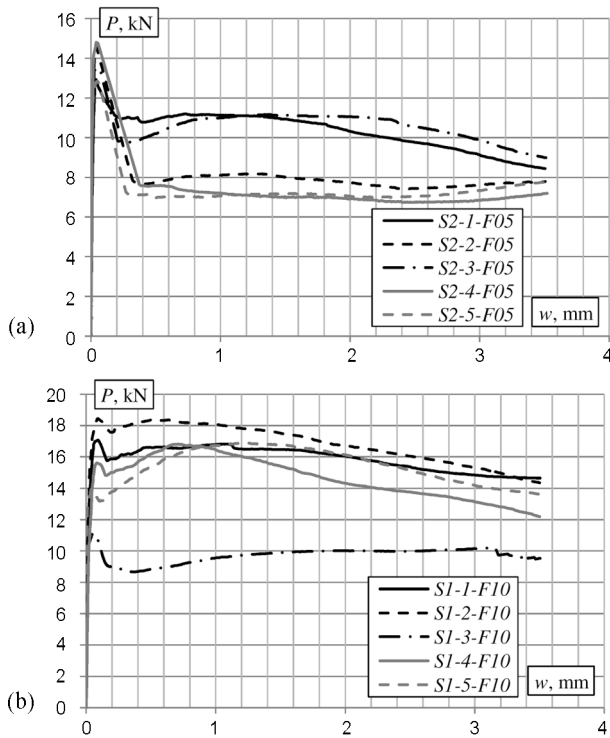


Fig. 5. Experimentally obtained load-crack opening width relationships for the test specimens with 0.5% (a) and 1.0% (b) fibre volumes

It is assumed that at the crack, the tensile strain approaches infinity, whereas the stress-strain relation before cracking defines the strain at a distance besides the crack. To determine tensile strain of concrete at a certain loading step, a length, further called influence length, is considered. For the beam, shown in Figure 6, the influence length over which the stresses are distributed due to the opening crack is taken equal to two times the height of the tensile zone (Dupont 2003).

The unknown position of the neutral axis is obtained from expression of residual stresses by simultaneously solving equilibrium equations of axial forces and bending moments:

$$\sigma_{r,n} = 0.5E \left[\varepsilon_n^2 (\bar{h} - 1) - \varepsilon_{cr}^2 \right] - \sum_{i=2}^n \sigma_{r,i-1} (\varepsilon_{i-1} - \varepsilon_{i-2}) = \frac{P_n L \varepsilon_n^2}{2b y_n^2} - \frac{2E}{3} \left[\varepsilon_n^3 (\bar{h} - 1) + \varepsilon_{cr}^3 \right] - \sum_{i=2}^n \sigma_{r,i-1} (\varepsilon_{i-1}^2 - \varepsilon_{i-2}^2) \quad (3)$$

$$\varepsilon_{n-1} + \varepsilon_n$$

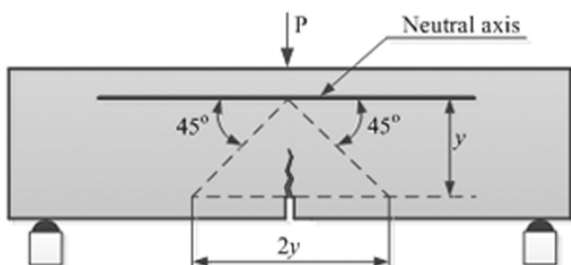


Fig. 6. Definition of the influence length for a beam

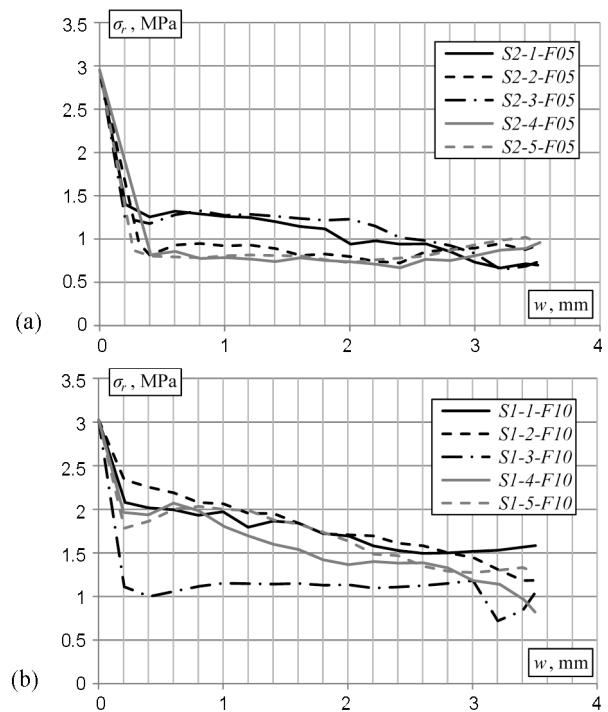


Fig. 7. Determined residual stress-crack width relationships for the test specimens with 0.5% (a) and 1.0% (b) fibre volumes

Residual stresses and, therefore, crack width can be found from equilibrium of axial forces:

$$0.5E \left[(\bar{h} - 1)^2 - \varepsilon_{cr}^2 y_n^2 / \varepsilon_n^2 \right] - \sum_{i=1}^n \sigma_{r,i} (\varepsilon_i - \varepsilon_{i-1}) = 0; \quad (4)$$

$$\bar{h} = h_{sp} / y_n, \quad \varepsilon_{cr} = f_{ct} / E, \quad \varepsilon_n = w_n / (2y_n), \quad \varepsilon_0 = \varepsilon_{cr}$$

In Eqns (3) and (4), n is the loading step under consideration; $\sigma_{r,n}$ is the residual stresses, ε_n is the deformation of element, w_n is the crack width, y_n is the position of neutral axis, and P_n is the load at n -th step; E and f_{ct} are the deformation modulus and tensile strength of SFRC; h_{sp} is the depth of the notched section; L and b are the span and the width of specimen, respectively.

It should be mentioned that with increase in loading, the position of the neutral axis changes; thus, it should be identified for every loading step, considered in the analysis. Obtained residual stress-crack width relationships for the beams with 0.5 and 1.0% of fibres are given in Figures 7a and 7b, respectively.

4. Numerical modelling

4.1. Standard beams

This section considers standard SFRC beams tested in the current study (Section 2). Adequacy of the residual stress-crack width (σ_{r-w}) relationships obtained by Naaman (2003) and RILEM (2002a) methods and derived by the proposed technique was verified using nonlinear finite element (FE) program ATENA. Figure 8 presents the test beam modelled employing the derived σ_{r-w} diagrams as constitutive models for SFRC in tension. Simulated load-crack width ($P-w$) relations for the beams with 0.5 and

1.0% of fibres are given in Figures 9 and 10, respectively. As it can be seen, the horizontal parts of $P-w$ curves, representing the residual loads, has an error up to 10, 25, and 65% for the proposed technique and for the RILEM and Naaman methods, respectively.

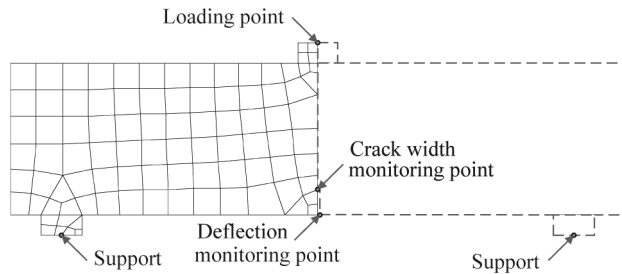


Fig. 8. Model of experimental beam using FE software ATENA

4.2. Full-scale beams

This section considers full-scale SFRC beams with steel bar reinforcement, tested by the authors. Two beams were

selected from the test program reported by Gribniak *et al.* (2012) with similar concrete mixture and fibre volumes as the standard specimens (Section 2). The specimens were reinforced with three 10 mm bars, resulting in tensile reinforcement ratio of 0.3%. Main parameters of the beams are listed in Table 1, where V_f is the volume content of fibres in the concrete mixture; f'_c is the $\varnothing 150 \times 300$ mm cylinder strength; E_s is the elastic modulus of the bar reinforcement.

Investigation of FE mesh dependence on modelling results of flexural members has shown that models of reinforced concrete beams having 6–8 finite elements per height demonstrate sufficient accuracy (Gribniak *et al.* 2010). In the current study, six elements per height were taken for FE modelling (Fig. 11). Due to symmetry conditions, only half of the beam was modelled. Isoparametric quadrilateral FE with 8 degrees of freedom and four integration points were used. Bar reinforcement was modelled by truss elements. The tension-stiffening effect was included in the FE model using principles of fracture mechanics (Gribniak *et al.* 2013a).

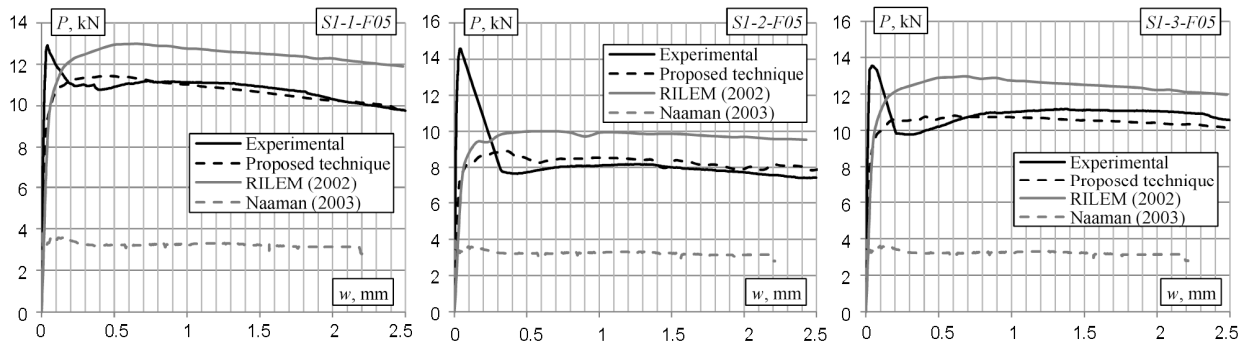


Fig. 9. Comparison of experimental and calculated $P-w$ curves for the experimental beams with 0.5% volume of fibres

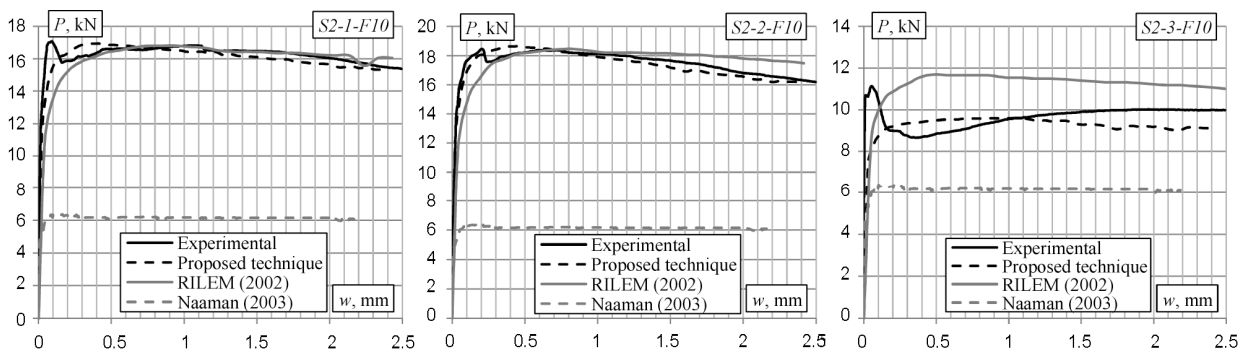


Fig. 10. Comparison of experimental and calculated $P-w$ curves for the experimental beams with 1.0% volume of fibres

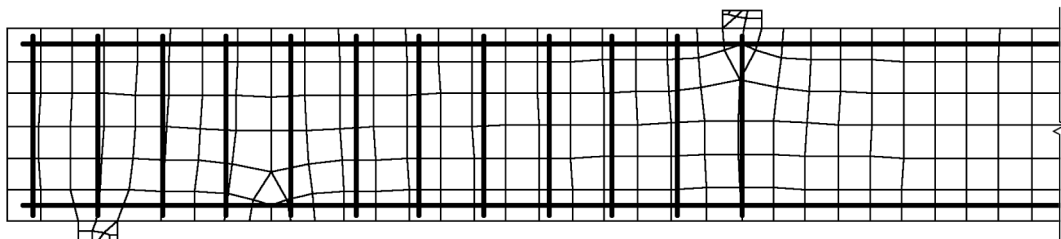


Fig. 11. Finite element meshing of a beam

Table 1. Geometric and material parameters of the full-scale beams

Beam	h , mm	d , mm	b , mm	a_{s2} , mm	A_{s1} , mm ²	A_{s2} , mm ²	V_f , %	f'_{cs} , MPa	E_s , MPa
S3-1-F05	302	278	278	29	235.4	55.9	0.47	55.6	202.8
S3-1-F10	300	276	279	23	235.4	55.9	1.02	48.0	202.8

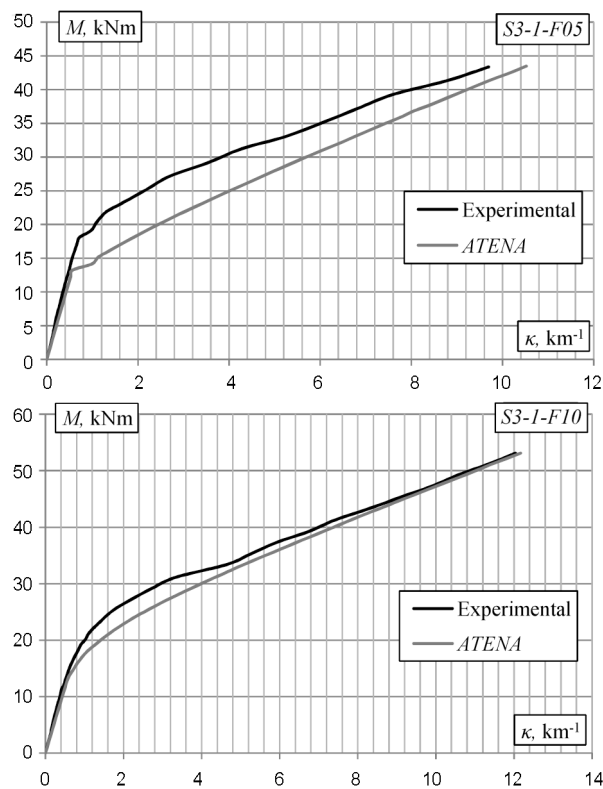


Fig. 12. Deformation analysis results on full-scale beams

Modelling results are presented in Figure 12. It can be observed that curvature is obviously overestimated for the beam with 0.5% of fibres. This is mainly due to the non-homogeneous distribution of the fibres in the standard concrete specimens, used for the constitutive modelling of SFRC. The employed material law ($\sigma_{fr}-w$ relationship) was derived for a single section, which may not necessarily contain sufficient number of fibres and may not represent the average deformation behaviour of full-scale beam. Furthermore, the tension-stiffening effect was included into FE model in “default” manner, but not assessed from the standard SFRC beams (without bar reinforcement) that may become another source of overestimated deformations of the full-scale beam. Increasing volume of fibres makes concrete more homogeneous. Increased residual strength of SFRC relatively reduces the effect of tension-stiffening (Fig. 1). Therefore, the assumed material law more adequately represents defor-

mation behaviour of full-scale beam with 1.0% of fibres than for 0.5% (Fig. 12).

Conclusions

The paper deals with experimental and theoretical investigation of the post-cracking behaviour of steel fibre reinforced concrete (SFRC). Ten standard beams equipped with notch and containing fibre contents of 0.5 and 1.0% by volume were tested under a three-point bending scheme. For the determination of the residual strength of SFRC in tension, a simple method based on the general principles of material mechanics was proposed. The main advantage of the proposed technique (in comparison with the methods provided by the RILEM or Naaman) is its capability to calculate residual stresses and, thus, crack width at any loading step.

To verify the obtained residual stress-crack opening relation a nonlinear finite element analysis program ATENA was utilized. The proposed constitutive analysis technique was found to be accurate determining the residual stresses in SFRC. The numerical simulation of full scale SFRC beams with steel bars has revealed importance of the sufficient amount of fibres in concrete and, therefore, residual strength on deformation prediction results: with increased volume of fibres (from 40 to 80 kg/m³) the residual strength becomes the governing criterion for the finite element modelling.

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