



## INFLUENCE OF LONGITUDINAL REINFORCEMENT ON DEVELOPMENT OF NORMAL CRACKS

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**Abstract.** Using a model of normal crack propagation in reinforced concrete members, an analytical expression is presented for the stress intensity factor defining the intensity of stress in area at the crack tip. This factor is in linear relationship to the stress in tensile reinforcement, which can be determined by different methods. The analysis performed indicates that there is a direct relationship between the stress intensity factor and normal crack depth, when a stable and chaotic propagation of cracks in the initial cracking stage of the member terminates. In addition, expression of this relationship depends significantly on reinforcement ratio of underreinforced beams.

**Keywords:** stresses, tensile reinforcement, normal cracks, stress intensity factors.

### 1. Introduction

Many reinforced concrete structures in operation are with cracks. The greatest stresses in their tensile reinforcement are in normal cracks since there the greatest quantity of energy is consumed for destruction of concrete structure at crack tips. The energy used for formation of new surfaces of cracks can be equivalently replaced the stress intensity factor defining the stress in area at micro-crack tip. According to proposed model for normal crack propagation in reinforced concrete beam [1], one tip of normal crack coincides with the centre of tensile reinforcement restraining the crack. Stress in tensile reinforcement increases with propagation of crack in concrete towards the neutral axis of reinforced concrete member and value of stress intensity factor  $K_I$  increases as well. As soon as this value reaches the critical one,  $K_{IC}$ , failure of underreinforced concrete member in normal crack starts, ie, yield stress value in tensile reinforcement occurs.

Many theoretical and experimental investigations have been performed in formation and propagation of cracks in cement concrete using criteria of the fracture theory during the last 20–30 years [1–4]. But there are comparatively quite few investigations of propagation of cracks in reinforced concrete (especially with clarification of the role of tensile reinforcement for intensity of cracks propagation), in which fracture mechanics of solids is used [1, 5–8].

In this article a method for calculation of stress intensity factor for reinforced concrete is proposed, in

which the main role in stabilisation of propagating cracks in reinforced concrete beams is performed by tensile reinforcement. With relevant reliability regulation of the critical stress intensity factor  $K_{IC}$  it is possible to determine safe limits (when  $K_I \leq K_{IC}$ ) for operation of reinforced concrete member.

### 2. Methods for determination of stress intensity factor

According to the model of normal crack propagation in reinforced concrete member [1] crack in concrete propagates in one direction only, towards the neutral axis. Propagation of crack is restrained by tensile reinforcement, which is conventionally assumed uniformly distributed along the width of cross-section as a solid strip. According to this model, stress intensity factor

$$K_I = 1,6\sigma_s \frac{\sqrt{t}}{a}, \quad (1)$$

where  $\sigma_s$  – reinforcement stress;  $t = (A_s/b)$  – thickness of solid reinforcement strip;  $\alpha \left( = \frac{E_s}{\varphi E_{cm}} \right)$  – reinforcement and concrete modular ratio ( $\varphi \approx 0,85$ ).

In this case factor  $K_I$  (and its critical value  $K_I$ ) is not a parameter for reinforcing steel as for a separately taken material. The factor evaluates influence of tensile reinforcement and indirectly of transverse strains of concrete in compression zone on restraint of crack in rein-

forced concrete member.

Reinforcement stress in normal crack [1]:

$$\sigma_s = \left[ \frac{(M_{tot} - \Delta M)[0,75(h_{cr} - d_1) + h_{ct}]}{I_1} - \frac{N_{tot}}{A_1} \right] \cdot \frac{\pi \alpha}{4\omega} \sqrt{(h_{cr} - d_1)/t}. \quad (2)$$

Putting of stress  $\sigma_s$  value in formula (1), results in such an expression for factor:

$$K_{I,1} = 1,26 \left[ \frac{(M_{tot} - \Delta M)[0,75(h_{cr} - d_1) + h_{ct}]}{I_1} - \frac{N_{tot}}{A_1} \right] \cdot \frac{\sqrt{h_{cr} - d_1}}{\omega}. \quad (3)$$

Here  $M_{tot} = M - Pe_p$ ,  $N_{tot} = P \pm N$  ( $P$  ir  $N$  – precompression and axial forces in the member; compression force  $N$  is taken with plus sign;  $e_p$  – eccentricity of force  $P$  in relation to cross-sectional centre);  $\Delta M = N_{tot}(y - y_1)$  (Fig 1);  $\omega = 125\rho$ , when reinforcement ratio  $\rho = A_s/(bd) < 0,008$  and  $\omega = 1$ , when  $\rho \geq 0,008$ ;  $A_1$  ir  $I_1$  – area and second moment of equivalent concrete cross-section, depth of which is  $h_1$ , in relation of cross-section centre superposed with the neutral axis; values  $d_1$ ,  $y$ ,  $y_1$ ,  $h_{cr}$ ,  $h_{ct}$  see in Fig 1.

Depth of design cross-section  $h_1$  is determined by the condition:

$$h_{cr} - d_1 + h_{ct} = S_1/A_1, \quad (4)$$

where  $S_1$  – the first moment of area  $A_1$  in respect to tensile reinforcement centre.

Depth of concrete zone in tension above crack  $h_{ct}$  can be determined from empirical formula [1]:

$$h_{ct} = 0,00012d_1^3 \sqrt[3]{d_s} \lambda_1 \frac{h_{cr}}{w}, \quad (5)$$

where  $d_s$  – reinforcement bar diameter, mm;  $w$  – crack width at the level of reinforcement centre;  $\lambda_1$  – coeffi-

cient for evaluation of effect of tensile flanges in cross-section to the depth  $h_{ct}$ , of bond between reinforcement and concrete etc [1]. Average deviation of values  $h_{ct}$  calculated according to (5) from experimental ones is (– 0,4 %), standard deviation from the average value – 26 %. Formula (5) is valid, when ratio  $d_1/h = 0,08 \div 0,22$  and reinforcement diameter  $d_s \leq 25$  mm.

Stress in tensile reinforcement can be calculated by the method proposed by A. Rozenbliumas [7], in which functional relationship between stresses, acting forces, measured normal crack depth and geometrical parameters of cross-section was established and approved experimentally, when crack depth  $h_{cr,act} > h_{f1}$ :

$$\left. \begin{aligned} h_{ct} &\approx \frac{(I - S h_{cr,act}) - (S - A h_{cr,act})d_1}{\left(\frac{M}{2f_{ct}} + S\right) - \left(\frac{N_{tot}}{2f_{ct}} + A\right)d_1}, \\ \sigma_s A_s &= \left[ \frac{S - A h_{cr}}{h_{ct}} - \left(\frac{N_{tot}}{2f_{ct}} + A\right) \right] 2f_{ct} \end{aligned} \right\} \quad (6)$$

here  $M$  – bending moment in respect of the edge of cross-section subjected to the greatest tension;  $A$  – cross-sectional area of concrete excluding area of concrete within the measured depth of the crack  $h_{cr,act}$ ;  $S$  and  $I$  – the first and the second moments of area  $A$  in respect to the edge of the cross-section in the greatest tension;  $f_{ct}$  –

tensile strength of concrete;  $\alpha = \frac{E_s}{0,85 E_{cm}}$ . In formulae (6) slip between reinforcement and concrete and strains of concrete in cracked intervals of a beam are taken in to account.

Using formulae (1) and (6) factor

$$K_{I,2} = \frac{3,2f_{ct}\sqrt{t}}{A_s \alpha} \left( \frac{S - A h_{cr,act}}{h_{ct}} - A - \frac{N_{tot}}{2f_{ct}} \right). \quad (7)$$

C. Bosco and A. Carpinteri [6] proposed such formula for determination of stress intensity factor:

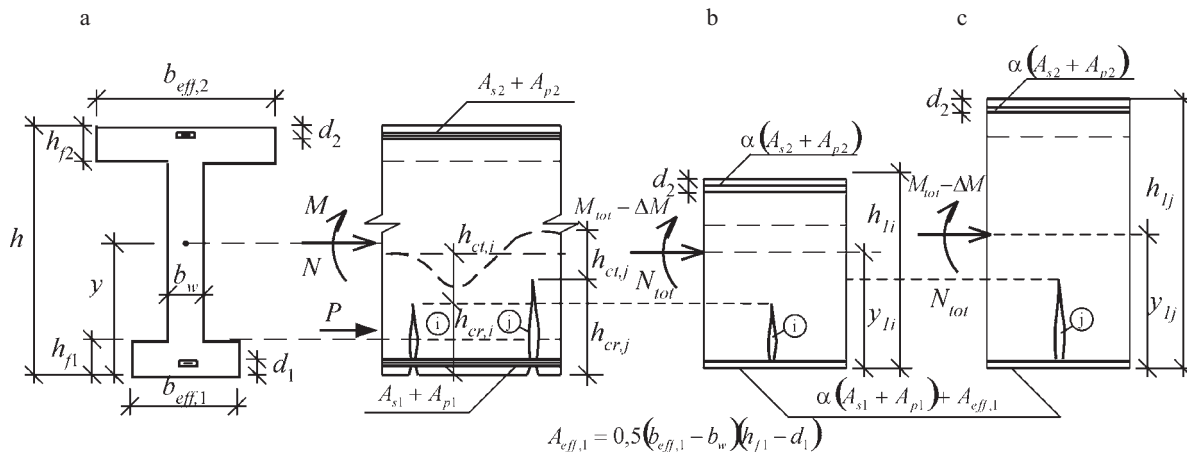


Fig 1. Diagrams for determination of stress intensity factor: a – cross-section of member and its interval with cracks; b – design case, when  $h \geq h_1$ ; c – design case, when  $h < h_1$

$$K_{I,3} = K_{IM} - K_{Ip} = \frac{M}{h\sqrt{h \cdot b}} y_m(\alpha_1) - \frac{\sigma_s A_s}{\sqrt{h \cdot b}} y_p(\alpha_1, \beta_1), \quad (8)$$

$$\left. \begin{aligned} h_{cr} + h_{ct} &\approx h - 1,25\xi h_0, \\ h_{ct} &\approx 2\alpha(h_{cr} + h_{ct})f_{ct}/f_y, \\ f_{ct} &\approx 5f_{c,cube}/(45 + f_{c,cube}), \\ E_{cm} &= 55000f_{c,cube}/(27 + f_{c,cube}). \end{aligned} \right\} \quad (9)$$

here  $M$  – bending moment in the zone of the beam in pure bending;  $y_m(\alpha_1)$  and  $y_p(\alpha_1, \beta_1)$  – functions to allow for geometrical parameters of the crack and cross-section ( $\alpha_1 = h_{cr}/h$ ,  $\beta_1 = d_1/h$ ). Restraining effect of tensile reinforcement on propagation of the crack is evaluated by the factor  $K_{Ip}$ .

### 3. Analysis of factor $K_I$ variation

Bond between reinforcement and concrete is one among other effects influencing propagation of cracks in reinforced concrete members. In calculation of stress intensity factor  $K_{I,1}$  according to formula (3) this effect is evaluated indirectly via normal crack parameters  $h_{cr}$  and  $w$  (see formula (5)). Damages of this bond (slip between reinforcement and concrete) can result in changes of factor  $K_I$  value. Data of known investigations of cracked intervals of reinforced concrete beams, among them and these of [9], proved that damages to bond between reinforcement and concrete are not significant for calculated values of  $\sigma_s$ . Influence of bond quality is greater in the initial stage of cracking of members, especially when their reinforcement ratios are small.

Theoretical analysis of stress intensity factors was performed using results of investigations [9, 10], which include underreinforced beams of rectangular cross-section. Concrete and reinforcement strengths, reinforcement ratios and magnitude of normal cracks  $\lambda = 1 - (h_{cr} - d_1)/h$  of these beams varied in very large ranges.

In case when crack width  $w$  data were not available in investigations [9], depth of concrete tensile zone above the crack  $h_{ct}$  for formulae (3) or (7) was calculated using formulae (6). It has been proved by experiments that calculated  $h_{ct}$  and experimental  $h_{ct,act}$  values are sufficiently close.

Values of stress  $\sigma_s$  in formula (8) were taken the same as those in calculating of  $K_{I,1}$  values.

Slip between reinforcement and concrete was taken into account in calculations of  $K_{I,1}$  values for reinforced silicate concrete beams by the means of multiplication of stress value in prestressed reinforcement by coefficient 0,67 [11].

For analysis of factor  $K_I$  the following data of investigations [10] were used: cross-sectional parameters of beams, relative compression zone depth  $\xi$  (related to rectangular stress diagram in stage of incipient failure of the beams), concrete cube strength  $f_{c,cube}$ , yield stress of reinforcement  $f_y$  and values of bending moments in beams in the stage of their incipient failure. In this case the lacking values  $h_{cr}$ ,  $h_{ct}$ ,  $E_{cm}$  or  $f_{ct}$  were determined from such expressions:

Values of  $f_{c,cube}$  are taken in MPa.

Values of factor  $K_{I,1}$  or  $K_{I,2}$  coincide very well for beams reinforcement of which is not prestressed ( $P = 0$ ); greater difference between these values is obtained for prestressed concrete beams ( $K_{I,1} > K_{I,2}$ , except for beam No 5, which failed in shear).

Values of factor  $K_{I,3}$  in average are about 1,32 times greater than those of  $K_{I,1}$  (Table), ie resistance of reinforced concrete beams to propagation of normal cracks according to the method [6] is less when values of acting stresses  $\sigma_s$  are equal in both types of beams. Differences between values of factors  $K_{I,1}$  and  $K_{I,3}$  increase not significantly with stress in longitudinal reinforcement.

Propagation of normal cracks is restrained not only by tensile reinforcement. Their propagation is impeded, especially in the stage close to failure of the member, by transverse strains of concrete in compression zone. Due to these strains microcracks directed along the beams occur, which later merge and develop in longitudinal macrocracks in the stage before final failure of the beam. Propagation of normal cracks slows down and increase of stress in tensile reinforcement intensifies with approach of state of stress in concrete member to the limit of failure. Factor  $K_{I,3}$  according to (8) evaluates this effect only partially, since value of  $K_{IM}$  is determined from tests of plain concrete beams with the zone of pure bending. In bending members of plain concrete there is not enough time for transverse of concrete strains to come into play and therefore their influence on propagation of normal cracks is less. According to the model presented in [1], propagation of crack is restrained not only by tensile reinforcement but also by transverse strains of concrete in compression zone. In this way it is possible to explain the increase in difference between values of factors  $K_{I,1}$  and  $K_{I,3}$  with approach to the failure limit of the member.

Nevertheless, it suffices to increase the factor  $K_{Ip}$  in formula (8) by 3–7 % (or adequately to reduce value of  $K_{IM}$ ) for values of  $K_{I,3}$  to become equal to these of  $K_{I,1}$ .

There is a relationship between parameters  $K_I/b$  and 1 (Fig 2), here  $K_I = K_{I,1}$  and  $\beta = 0,02f_y\sqrt{t}$ .

Relationship between parameters  $K_I/b$  and 1 can be expressed as follows:

$$K_I = 11,37\beta(1 - 1,3\lambda^{1,63}), \quad (10)$$

here  $K_{IC} \geq K_I \geq 70\beta(1 - 0,2\rho)/\alpha$ .

Value of factor of stress intensity  $K_I$  increases with normal crack depth and in the stage of failure of beams, the critical value of the factor is attained ( $K_I/\beta \approx 8 - 11$  for beams of rectangular cross-section [10], Fig 2). The

Stress intensity factors  $K_I$

Beam No	$b \times h$ , m	$\rho$ , %	$f_{cs}$ MPa	$f_y$ MPa	$M$ , kNm	$P$ , kN	$K_I$ , MPa m <sup>1/2</sup>			$\frac{K_{I,1}}{K_{I,2}}$	$\frac{K_{I,1}}{K_{I,3}}$	Notes
							$K_{I,1}$	$K_{I,2}$	$K_{I,3}$			
1	0,2×0,4	0,54	0,90	445	14,9	0,0	0,92	0,91	1,03	1,01	0,89	Calculation examples; parameters $h_{cr}$ and $h_{cr}$ calculated according to [7]
					29,8		1,82	1,89	2,50	0,96	0,73	
					49,6		3,07	3,09	4,23	0,98	0,73	
34,3		1,37			1,38		1,73	0,99	0,79			
68,5		2,75			2,86		3,63	0,96	0,76			
114,2		4,51			4,78		6,64	0,94	0,68			
2		1,56	1,20	1,20	445	49,4	1,96	2,05	2,56	0,96	0,77	
						98,8	3,95	3,94	5,49	1,00	0,72	
						164,6	6,64	6,32	9,10	1,05	0,73	
3		2,53	1,20	1,20	445	86,4	2,38	1,76	–	1,35	–	
	106,4					3,21	2,68	–	1,20	–		
	126,4					3,96	3,56	–	1,11	–		
4	0,18×0,5	1,80	2,50	588	146,4	4,64	4,47	–	1,04	–		
					25,0	5,16	5,72	–	0,90	–		
					31,3	5,80	7,40	–	0,78	–		
5	0,08×0,3	1,18	3,57	1194	6,3	2,11	2,25	–	0,94	–	According to data of [7]; experimental $h_{cr,act}$ values	
					7,3	2,46	2,57	–	0,96	–		
6	0,08×0,18	2,03	1,50	286	29,4	0,56	0,45	–	1,24	–	According to data of [7]; Reinforced silicate concrete beams; $h_{cr,act}$ values applied	
					47,2	0,98	0,70	–	1,40	–		
7	0,105×0,185	0,88	1,65	588	7,9	0,96	0,94	–	1,02	–		
					11,0	1,35	1,33	–	1,02	–		
8	0,105×0,188	1,90	1,67	358	8,8	0,72	0,77	–	0,94	–		
					13,9	1,15	1,24	–	0,93	–		
9	3,22	1,67	358	358	13,9	1,15	1,24	–	0,93	–		
					13,9	1,15	1,24	–	0,93	–		
10	0,15×0,3	2,42	2,12	356	83,8	5,31	Average:		1,03	0,76	According to data of [8]	
11		0,49	2,82	354	24,5	2,63						
12		0,51	2,86	355	25,3	2,71						
13		1,50	2,62	362	55,6	3,74						
14		2,50	2,62	363	83,8	5,50						
15		1,26	3,45	354	50,8	3,21						
16		2,65	3,31	366	83,8	5,76						
17		3,75	3,50	358	133,3	6,92						
18		0,52	2,30	1043	62,8	7,69						
19		0,52	3,45	1040	77,8	8,12						
20		1,03	3,34	1054	124,3	9,48						
21	2,51	3,37	835	161,8	13,02							

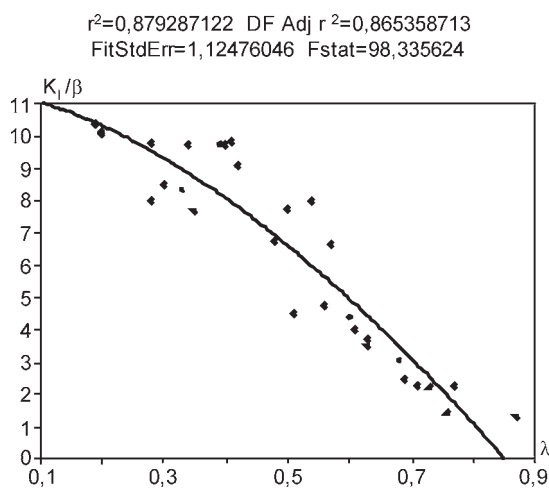


Fig 2. Relationship between factor  $K_I$  and the depth of normal crack  $h_{cr}$  (according to data of [9, 10])

limits of validity for formula (10) indicate that value of factor  $K_I$  depends on reinforcement ratio  $r$  of under-reinforced beams. Interval of application of formula (10) decreases with reinforcement ratio of beams. However, relationship (10) valid for beams up to their failure stage (when  $\sigma_s = f_y$ ) gives an opportunity to evaluate danger of open normal cracks for existing concrete structures.

#### 4. Conclusions

1. Different from plain concrete structures, propagation of normal cracks in reinforced concrete structures is stabilised by tensile reinforcement and transverse strains of concrete in compression zone. Therefore propagation of such macrocracks with sufficient accuracy can be controlled by the means of one general for reinforced concrete stress intensity factor  $K_I$  expressed via tensile reinforcement stress  $\sigma_s$ , reinforcement ratio  $\rho$  and location of reinforcement in cross-section of the member. Resistance of tensile concrete to propagation of the crack, in comparison with just mentioned effects restraining propagation of crack, is not significant.

2. Stress intensity factor  $K_I$  in underreinforced concrete beams can be calculated by (3) or (7) formulas. Factor  $K_I$  can be approximately calculated more simply by the means of established direct relationship between values of  $K_p$ ,  $h_{cr}$ ,  $t$ ,  $f_y$  and  $a$  (formula (10)). Application of such type of relationships is convenient for assessment of existing reinforced concrete structures with cracks.

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## IŠILGINĖS ARMATŪROS ĮTAKA NORMALINIŲ PLYŠIŲ PLITIMUI

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Santrauka

Pagal normalinio plyšio plitimo modelį gelžbetoniniame elemente galima apskaičiuoti tempiamosios armatūros įtempius. Pateikiama įtempių intensyvumo koeficiento, nusakančio įtempių lauko intensyvumą, analizinė išraiška. Šis koeficientas tiesiogiai proporcingas tempiamosios armatūros įtempiams. Įtempių intensyvumo koeficiento reikšmės buvo tarpusavyje palygintos ir pagal skirtingas metodikas apskaičiuoti tempiamosios armatūros įtempiai. Analizė parodė, kad egzistuoja aiški priklausomybė tarp įtempių intensyvumo koeficiento ir normalinio plyšio aukščio (ilgio), kai baigiasi nestabilus plyšių plitimas elemento pleišėjimo pradžioje. Be to, šios priklausomybės išraiškai didelės reikšmės turi neperarmuotų sijų armavimo koeficientas.

**Raktažodžiai:** įtempiai, tempiamoji armatūra, normaliniai plyšiai, įtempių intensyvumo koeficientai.

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