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DAMAGE DETECTION USING PARAMETER DEPENDENT DYNAMIC EXPERIMENTS AND WAVELET TRANSFORMATION

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Abstract. The paper deals with a class of problems, where localised damage is detected using static and dynamic tests. Response of a structure is analysed employing discrete wavelet transformation as a tool for signal processing. The localised damage in beam structures was modelled as bending stiffness reduction. The efficiency of static and dynamic tests is studied. The minimal number of experimental measurement data and the precision of measurements required for the successful damage localisation is discussed by several numerical examples.

Keywords: structural identification, damage detection, wavelet transformation.

1. Introduction

The problem of localisation and estimation of structural damage is one of the most important engineering problems. It is connected with the assessment of structure safety and serviceability. This issue belongs to a wider class of identification problems, where unknown parameters of a system are determined by experimental tests. Structural identification has focused much attention in the last two decades.

First papers devoted to damage detection were based on the modal analysis of structural response. It was observed [1, 2] that damage results in stiffness reduction, increase of damping and decrease of natural frequency of a structure. An extensive review of damage identification techniques using information on changes in natural frequencies and shape modes of vibrations was presented [3]. Unfortunately, global static and dynamic responses are rather insensitive to localised damage and often are smaller than the variations resulting from inaccuracy in structure modelling. Hence, different ways of improvement of experiments were proposed. Variable location of support or additional concentrated mass enhancing structural response was proposed [4]. Optimization of loads with the aim to better exhibit the discrepancy between responses of the damaged and undamaged structures was presented [5–7].

New class of identification methods emerged with the development of the so-called soft methods: fuzzy sets, genetic algorithms and neural networks. Application of genetic algorithms in damage identification was presented in [8], whereas prospects of neural networks was discussed in [9]. In paper [10] fuzzy sets, genetic algorithms and neural networks were used simultaneously.

Entirely new method rooted in the mathematical theory of signal analysis emerged in the last years, namely, wavelet transformation. It has become one of the most promising tools in structural identification. The theory of wavelet transformation was developed by Daubechies [11], Mallat [12] and Chui [13]. Newland [14] showed the potential of this tool in vibration signal analysis. Wavelet transformation was applied to damage identification by Wang and McFadden [15]. Application of the method in the space domain was presented [16]. Analysis of transformation parameters was performed in [17], where the efficiency of wavelets in damage identification was demonstrated.

In this paper we continue discussion on application of wavelet transformation in structural damage identification. The attention will be focused on the identification efficiency in the analyses of static and dynamic structural responses. The problem of the number of measurement points and the influence of noise in measurements are discussed.

2. Problem formulation

We use beam, frame or plate models of damaged structures. The specific type of the structure does not make much difference, provided that we can receive the response for any action (not necessarily defined). Our main task is to detect localisation of damage in the structure, if such a damage exists. This localisation will be determined basing on signal analysis of structural response to different actions.

We consider numerical models of beams loaded by static and dynamic forces. In all examples the damage is modelled as local stiffness reduction at small, prescribed distance introduced at the point of existing damage. The application of damage model in the form of an elastic hinge was discussed by the authors in [17].

In the procedure of damage identification various structural responses are analysed, namely static displacements or the amplitudes of dynamic displacement and acceleration. The structural response represented in the form of a discrete signal is transformed using wavelet transformation. Theoretical background of this transformation was published in [11–14]. Basic information on discrete wavelet transformation, directly connected with the present study, will be provided in Chapter 3.

As a result of signal processing we expect evident disturbances of transformed signal to appear in the place of damage location. We assumed that the response of undamaged structure is not known. The aim of the paper is not limited to demonstration that the defect identification is possible. Crucial problem is how to achieve this goal with the minimum number of measurement points and with a specified noise level representing measurements inaccuracy inevitable in experiments.

3. Basis of wavelet transformation

Wavelet transformation is a method of decomposition of arbitrary signal *f(x)* into an infinite sum of wavelets at different scales according to the expansion:

$$f(x)=\sum_{j=-\infty}^{\infty}\sum_{k=-\infty}^{\infty}c_{j,k}W\left(2^{j}x-k\right), \tag{1}$$

where *W(x)* is a wavelet (mother) function. Integers j and k are dilation (scale) and translation (position) indices, respectively. The terms $c_{j,k}$ are numerical constants called wavelet coefficients.

When j is negative, $W(2^j x\text{-}k)$ can always be expressed as a sum of terms $\phi(x-k)$, providing

$$f(x)=\sum_{k=-\infty}^{\infty}c_{\phi,k}\phi(x-k)+\sum_{j=0}^{\infty}\sum_{k=-\infty}^{\infty}c_{j,k}W\left(2^{j}x-k\right), \tag{2}$$

where $\phi(x)$ is a scaling (father) function and $c_{\phi,k}$ is a new set of coefficients.

In order to set up a discrete wavelet transform algorithm, it is convenient to limit the range of the independent variable x to one unit interval so that *f(x)* is assumed to be defined only for $0 \le x \le 1$. Then, the variable x is non-dimensional. Additionally, assuming that *f(x)* is one period of a periodic signal, the wavelet expansion can be written in the form

$$f(x)=a_0\phi(x)+\sum_{j}\sum_{k}a_{2^j+k}W\left(2^{j}x-k\right). \tag{3}$$

The coefficients a_{2^j+k} represent the amplitudes of subsequent wavelets. The integer j describes different levels of wavelets starting from $j = 0$. Integer k specifies the number of wavelets at each level, so that it covers the range $k = 0$ to $2^j–1$.

The discrete wavelet transform, which is used in this paper, is an algorithm for computing coefficients a_{2^j+k} when *f(x)* is sampled at equally spaced intervals over $0 \le x \le 1$. Since the number of sampled values is limited, every function *f(x)* is approximated by $f_J(x)$ using $N = 2^J$ discrete values. Note that the scale indicator $j = 0, 1, \ldots, J–1$. Henceforth, keeping in mind (3), the discrete signal decomposition can be written as

$$f_J = f_\phi + f_0 + f_1 + \ldots + f_j + \ldots + f_{J-2} + f_{J-1} \tag{4}$$

or

$$f_J = S_M + D_M + D_{M-1} + \ldots + D_m + \ldots + D_2 + D_1, \tag{5}$$

where $m = J - j$.

Each component in signal representation provides information at the scale level j. The last terms in (4) and (5) corresponds with the most detailed representation of the signal (high-frequency oscillations). The preceding representations deliver the more and more rough information about the signal and correspond to the lower frequency oscillations.

In practice DWT requires neither integration, nor explicit knowledge of scaling and wavelet functions, generating transformation. Due to the character the above method of signal representation it is called multi-resolution analysis (MRA).

4. Numerical analysis

4.1. Static and dynamic structural responses

To compare the efficiency of damage identification for different signals representing static and dynamic structural responses, a propped cantilever beam model presented in Fig 1 was used.

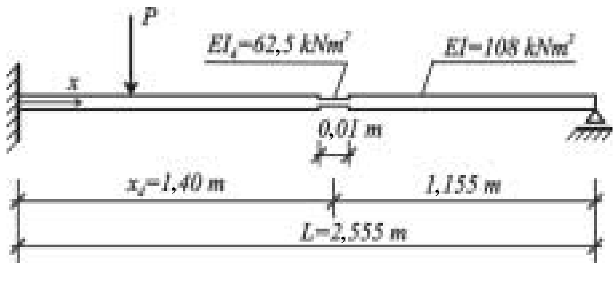

Fig 1. Beam structure with the damage modelled as stiffness reduction

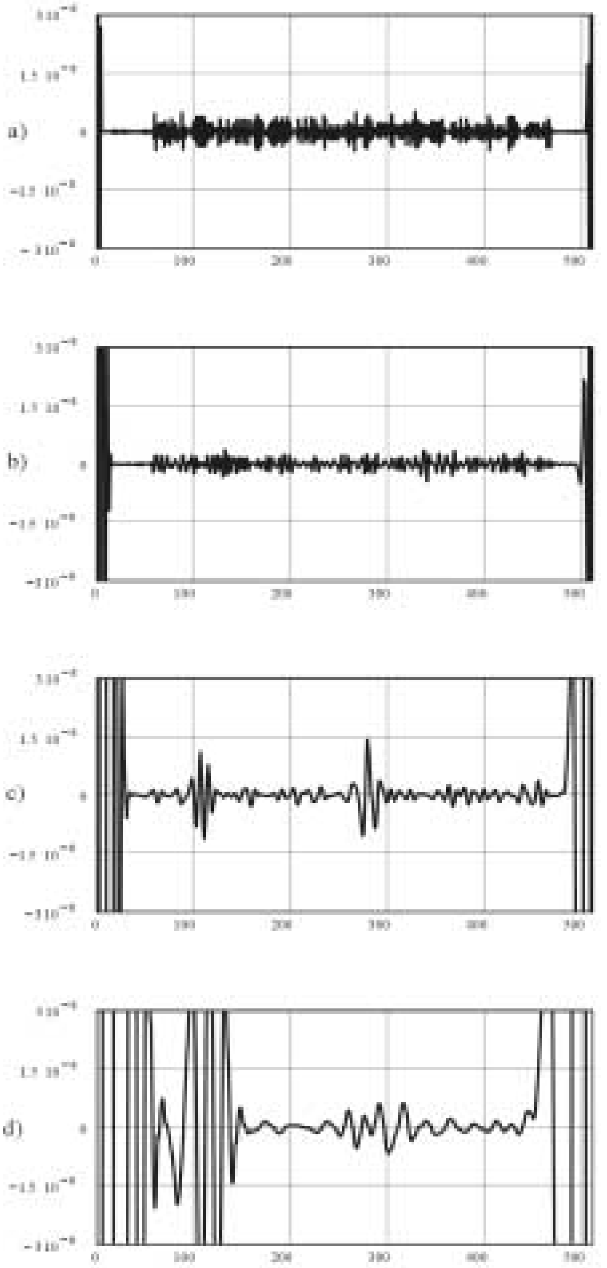

Fig 2. Wavelet transformation using wavelet daublet8 of static displacement signal for the force localisation $x = 0,5$ m: a) detail D_1, b) D_2, c) D_3, d) D_4

The properties of the structure are: bending stiffness EI = 108 kNm^2; damage localised at x =1,40 m measured from the clamped end; the damage is modelled as stiffness reduced to EI_d = 62,5 kNm^2 on the portion b = 0,01 m. All static and dynamic responses were computed using FEM (ABAQUS program). Two classes of problems were examined: static response to the concentrated force P=1 kN and steady-state vibrations excited by the dynamic force $P(t)=1cos(2\pi ft)$. Three frequencies of the dynamic excitation were assumed, namely: f_1= 10 Hz, f_2= 25 Hz and f_3= 80 Hz. The frequency f_2 is close to the first eigenfrequency of the system (32.81 Hz), whereas the frequency f_3 is near to the second one (106.50 Hz). In both, static and dynamic cases, the concentrated forces acting on the structure were localised in various points: x = 0,5 m, x = 1,0 m, x = 1,5 m and x = 2,0 m. Altogether, 16 analyses were performed.

As a structural response signal the static vertical displacements and the amplitudes of vertical displacements and accelerations were analysed. In each case the response signal was computed in 512 points uniformly distributed at the longitudinal axis of the beam. This data is treated as respective discrete signals in the space domain and is processed using wavelet transformation.

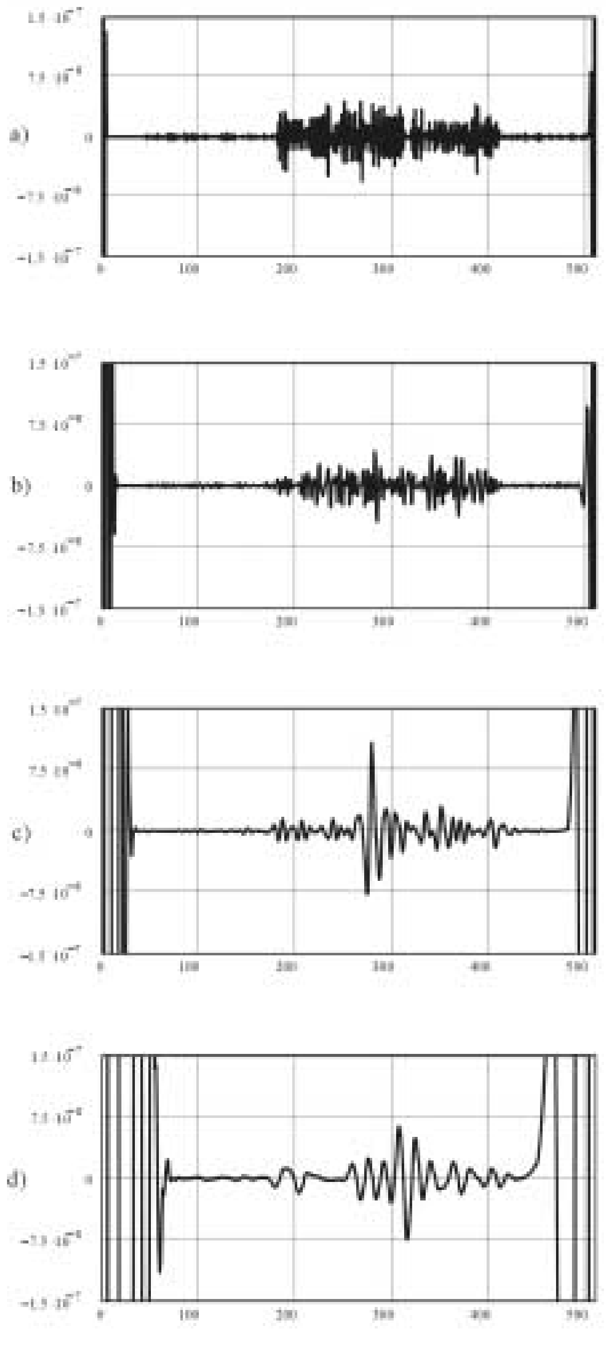

Fig 3. Wavelet transformation using wavelet daublet8 of static displacement signal for the force localisation $x = 1,5$ m: a) detail D_1, b) D_2, c) D_3, d) D_4

Usually, wavelets named daublet8 were used in this transformation. As a result of the transformation we obtained evident disturbances in the place where damage was localised in the most detailed signal representations, namely D_1, D_2 or D_3.

In Figs 2 and 3 details D_1, D_2, D_3, D_4 of the wavelet transformation of static response for the force localisation at the points $x = 0,5$ m and next $x = 1,5$ m are presented. Note in these figures that the damage (and also the force) localisation is demonstrated in the utmost evident way by the detail D_3. Defect is visible for both force positions, though greater effect was obtained for the case when the force was localised near the damage. It is worth to notice that in all graphs high disturbances at the ends of interval appeared. They increased with successive higher order signal representations. They result from the fact that the employed wavelets exceed the range [0,1]. Therefore in wavelet transform algorithm it was assumed that the analysed signal is periodic. Hence, overlapping of wavelets results in disturbances at boundaries.

To compare the efficiency of static and dynamic responses in damage identification, transformations of these responses are presented in Figs 3 and 4, respectively. The force P was localised at $x = 1,5$ m and vertical displacements were analysed. On the graphs 4a, b and c detail 3 for frequencies of excitation $f_1 = 10$ *Hz*, $f_2 = 25$ *Hz*, $f_3 = 80$ *Hz* is presented, respectively.

Fig 4 demonstrates that in this case damage was properly identified for the frequencies f_1 and f_2. The frequency f_3 has not detected the damage. Of course, it is connected with the specified damage position.

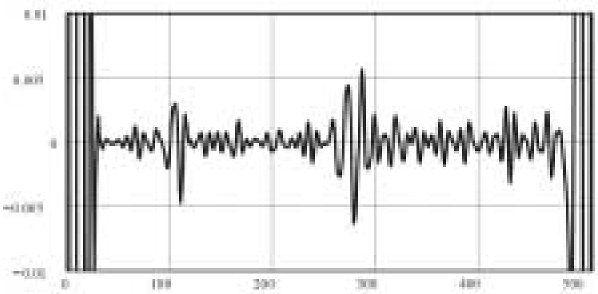

Fig 5. Detail 3 of acceleration amplitudes transformation for the force position $x = 1,5$ m and frequency: $f_2 = 25$ Hz

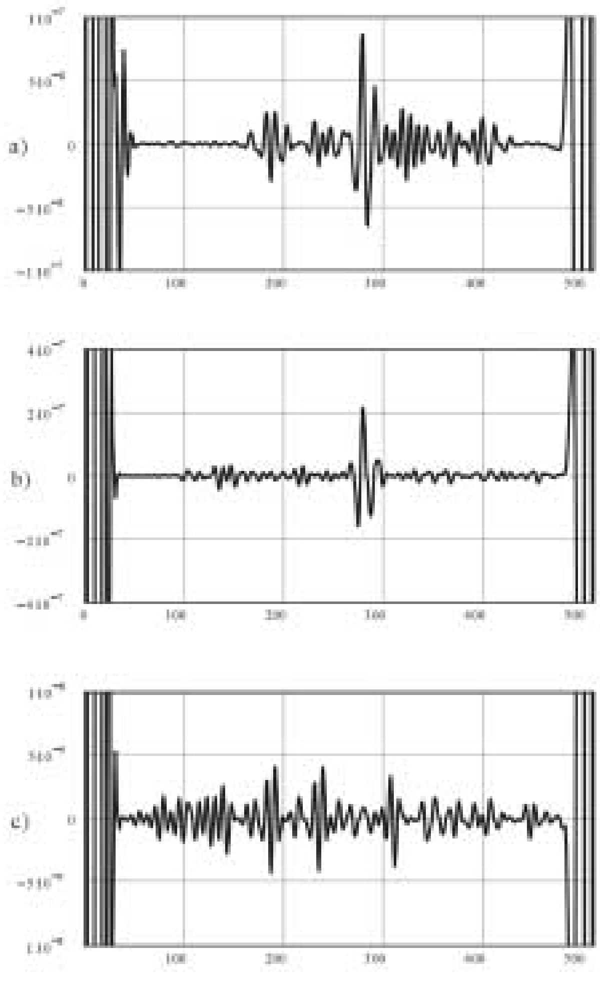

Fig 4. Detail 3 of displacement amplitudes transformation for the harmonic force localisation $x = 1,5$ m and frequencies: a) $f_1 = 10$ Hz, b) $f_2 = 25$ Hz, c) $f_3 = 80$ Hz

Next, we studied the usefulness of the structural response in the form of accelerations. Note, that it is often easier to measure in situ the accelerations than the displacements. Fig 5 illustrates the wavelet transform of the accelerations for the case f_2 identical to the case shown in Fig 4b.

Unfortunately, identification based on transformation of acceleration amplitudes has not brought expected results. It needs further studies.

Let us come back to the problem of a choice of excitation frequency. If we change our model so that the dynamic force is acting at the point $x = 2,0$ m and the damage will be defined at the point $x_d = 0,75$ m, proper damage identification will occur only for the frequency $f_3 = 80$ *Hz*. This phenomenon was shown in Fig 6.

Summarising the results of the above analyses we observe that damage is detected as well for static as for dynamic structural responses. The application of dynamic excitation provides more possibilities in planning the experiment. On the other hand, it can lead to some difficulties in the valuation of dynamic response.

4.2. Number of measurement points

One of the key problems in the damage identification is the prediction of the minimal number of measurements required for proper data processing. Fig 7 provides the first insight into this problem. On the successive graphs 7a, b, c, the wavelet transformations of the static response represented by 128, 64 and 32 uniformly distributed measurement points are depicted.

Basing on Fig 7 we can conclude that in our case the minimal number of measurements was 64. However, it is difficult to define arbitrarily the number of experi-

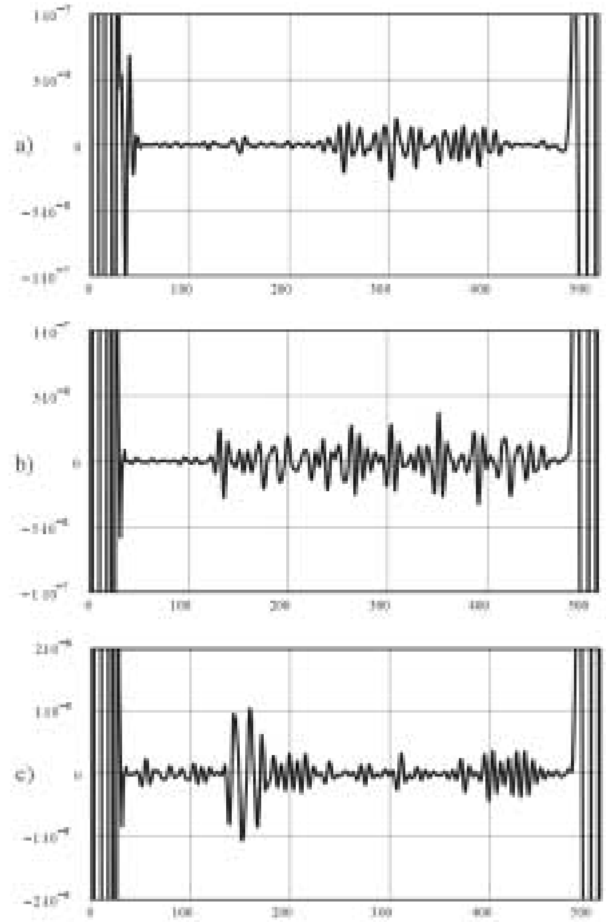

Fig 6. Detail 3 of displacement amplitudes transformation for the damage position $x_d = 0{,}75$ m, force localisation $x = 2{,}0$ m and frequencies: a) $f_1 = 10$ Hz, b) $f_2 = 25$ Hz, c) $f_3 = 80$ Hz

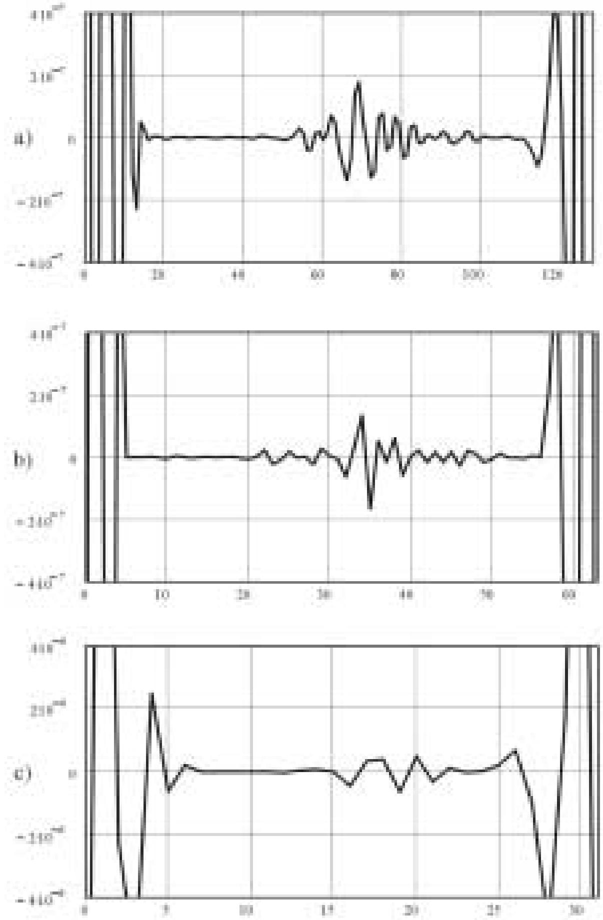

Fig 7. Wavelet transformation of static displacement signal for the damage position $x_d = 1{,}4$ m and force localisation $x = 1{,}5$ m; number of measurement points: a) 128, b) 64, c) 32

mental data, which is indispensable for damage identification. This number depends on many factors, for example, damage quantity measured relatively to the undammaged part of the structure. Future research will tend towards minimisation of the number of measurements and improvement of tools of signal processing.

4.3. Noise in measured data

Inevitable element of any experiment is measurement inaccuracy. In this part of the paper we want to quantify the level of measurement inaccuracy which made damage identification impossible. To model the inaccuracy we used white noise generator.

For the damage localised at the point $x_d = 1{,}4$ m and the force position $x = 1{,}5$ m, static displacements of the beam structure were analysed. The noise was superposed on the signal for N = 512 and 128 measurement points. Details of wavelet transformation are presented in Figs 8 and 9.

The performed analysis demonstrated that proper damage identification became practically impossible with the noise level $3 \cdot 10^{-7}$ m and $5 \cdot 10^{-7}$ m, respectively. It corresponded to measurement precision of the displacement in the damage position ($u_d = 1{,}5056 \cdot 10^{-3}$ m).

5. Concluding remarks

The application of wavelet transform to damage localisation was discussed in the paper by the way of several numerical examples. The analyses were based on both, static and dynamic responses of damaged structures. Information about undamaged structure was not used.

Presented examples showed that correct damage identification was possible using transformation of signals issued from both, static and dynamic experiments. Dynamic experiments provided more possibilities for damage identification, however, in this case the valuation of the experimental data could be more complicated than in the static case.

The examples demonstrated that damage was proper localized also with reduced number of experimental data and with a specified level of noise, representing measurement inaccuracy. The effectiveness of transformation

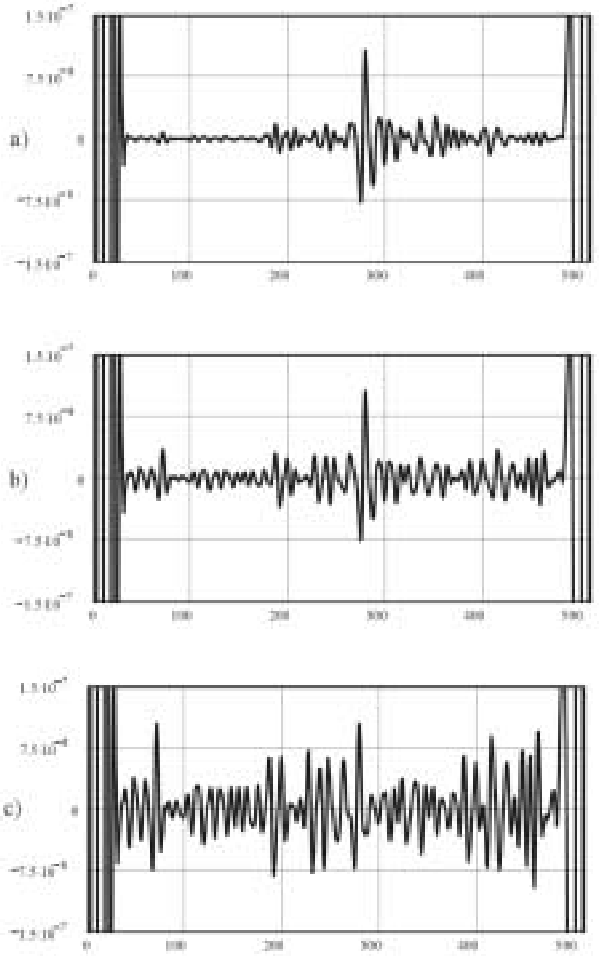

Fig 8. Wavelet transformation of static displacement signal for the damage position x_d = 1,4 m, force localisation x = 1,5 m and 512 measurement points; noise level a) $2 \cdot 10^{-8}$, b) $1 \cdot 10^{-7}$, c) $3 \cdot 10^{-7}$ m

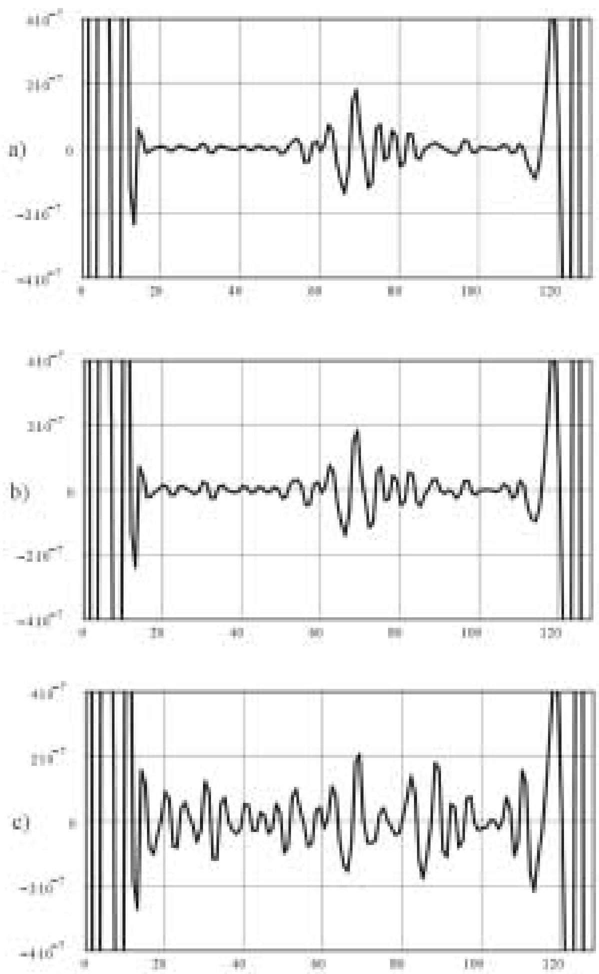

Fig 9. Wavelet transformation of static displacement signal for the damage position x_d = 1,4 m, force localisation x = 1,5 m and 128 measurement points; noise level a) $5 \cdot 10^{-8}$, b) $1 \cdot 10^{-7}$, c) $5 \cdot 10^{-7}$ m

with the use of common types of wavelets is limited by disturbances, which are induced at the ends of the space domain of the transformed signal.

The great advantage of wavelets is that information about the undamaged structure is not required in defect identification process. The examples discussed in the paper confirmed that wavelet transformation is a very promising tool in structural identification problems. The study will be continued in the direction of implementation of wavelet transformation to the identification of damage in real life engineering structures.

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