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NUMERICAL CREEP ANALYSIS OF REINFORCED CONCRETE FLEXURAL MEMBERS

I. Cypinas

1. Introduction

The concrete of tension zone makes significant contribution to flexural response of a reinforced concrete member even after formation of cracks. This con-tribution cannot be evaluated properly irrespective of the tension reinforcement and configuration of the tension zone. The tensional behaviour of plain concrete is de-scribed by a stress-strain curve with the falling branch. The deformation of a concrete fibre, taken separately, after attainment of peak tensile stress is unstable, but reinforcment bars and adjacent concrete of compression zone stabilises the process. The deformation process in the tension zone is very complicated, it includes the initial formation and coalescence of cracks into the major ones, and the slippage of reinforcement bars. This process can be described integrally, in terms of one-dimensional stress state, attributing to concrete certain non-linear stress-strain curve depending on the amount and distribution of the reinforcement. In publication [1] the authors propose certain exponential curve to describe decaying tensile response of concrete after occurrence of cracks. This curve depends on the reinforcement ratio and the diameter of reinforcement bars. The authors of publication [2] present the analytical expression

$$\sigma = \begin{cases} f_t' x, & x = \dfrac{\varepsilon}{\varepsilon_t'}, \quad \text{if} \quad x \le 1 \\ f_t' \dfrac{\beta x}{\beta - 1 + x^{\beta}}, & \text{if} \quad x > 1 \end{cases} \tag{1.1}$$

where f_t' is the tensile strength of concrete, ε_t'— the strain attained at f_t', β — an empirical parameter. The value of β can be computed by an empirical formula

$$\beta = \left(\frac{100 A_s}{b(h - x_{nt})}\right)^{0.366} \left(\frac{b(h - x_{nt})}{n \pi c d}\right)^{0.3436} \left(\frac{c}{s}\right)^{0.146} \tag{1.2}$$

based on experimental results. Here A_s is the area of the tensile reinforcement, b, h — width and depth of the rectangular cross-section, n — number of reinforcing bars, d — diameter of the reinforcing bars, c — concrete cover to reinforcement, s — reinforcement spacing, x_{nt} — the neutral axis depth, computed neglecting the tension in concrete. The value of x_{nt} has only small influence on the β, and so x_{nt} may be taken approximately.

The application of the expressions (1.1) and (1.2) for layer analysis of section in flexure implies the following assumptions: 1) the linear distribution of strains is retained, 2) the stress-strain relationship is the same for all layers of the tension zone.

The coupling between the creep and cracking phenomena is described by Z. P. Bažant and J.-C. Chern [3]. The problem is also considered by J.-C. Chern and A. H. Marchertas [4]. Their approach is to divide the total long-term strain $\varepsilon(t)$ into a linear creep component and a cracking component:

$$\varepsilon = \varepsilon^{c} + \xi . \tag{1.3}$$

The creep strain ε^{c} is governed by a linear integral-type constitutional relation, and depends on the time. The cracking strain ξ follows certain non-linear stress-strain relation, and it is assumed to be independent of time.

It is also assumed in this paper that the compression zone of the reinforced concrete member undergoes only linear creep deformations. This assumption is valid when the dead load does not exceed about a half of the total load. In the tension zone, however, the cracks occur unavoidably. Analytical relations for the interaction between creep and cracking are presented herein, and suitable numerical procedures are elaborated. Computer program has been written, and computational algorithm appeared to be efficient and numerically stable. This algorithm is intended for introduction into the major non-linear finite element code.

In his previous publication [5] the author has presented numerical modelling of a separate layer of concrete in tension. In author's report [6] the numerical model of reinforced concrete section in flexure was briefly described. The more detailed elaboration of the numerical model and its adjustment for the requirements of practical use is presented in this work.

2. Incremental creep relations

The basic creep relation is expressed in a linear integral form:

$$\varepsilon^c(t) = \sigma(t_0)\,J(t,t_0) + \int_{t_0}^{t} J(t,t')\,d\sigma(t'). \qquad (2.1)$$

The creep compliance function $J(t,t')$ is described analytically in corresponding design codes (see, for instance, EC2 [7]). In this paper, the latest propositions of Z. P. Bažant and his colleagues [8] are used. That relation is valid for the compression zone; in the tension zone, it is applicable until the stress reaches the tensile strength of concrete.

In the cracking stage the total long-time strain $\varepsilon(t)$ is seen as the sum of two components of equation (1.3). The instantaneous strain ε^s as an argument of function $\sigma = f(\varepsilon^s)$, determined for the short-time loading by the equation (1.1), can be divided analogically:

$$\varepsilon^s = \varepsilon^e + \xi. \qquad (2.2)$$

These components of strain are represented in Fig 1.

The main assumption in the approach, presented herein, is that the dependency between the cracking component of strain ξ and the stress σ is

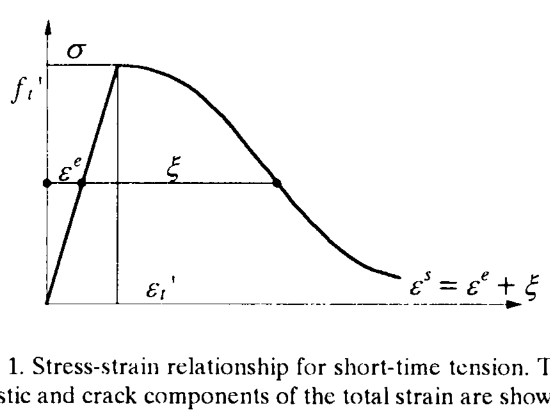

Fig 1. Stress-strain relationship for short-time tension. The elastic and crack components of the total strain are shown

the same for a sustained as well as for an instantaneous loading. Equating the quantity ξ in both sums (1.3) and (2.2), one will obtain the expression for an argument ε^s of function $\sigma = f(\varepsilon^s)$. The component ε^e, as it is seen in Fig 1, is a linear strain, $\varepsilon^e = \sigma/E_{sec}$, and so the expression for ε^s reads as

$$\varepsilon^s = \varepsilon - \varepsilon^c + \sigma/E_{sec}. \qquad (2.3)$$

The secant modulus here is $E_{sec} = f_t'/\varepsilon_t'$.

The main relationship of long-time cracking deformation can be formulated as

$$\left.\begin{aligned} &\sigma = f(\varepsilon^s), \\ &\varepsilon^s = \varepsilon - \sigma(t_0)J(t,t_0) - \int_{t_0}^{t} J(t,t')\,d\sigma(t') + \frac{\sigma(t)}{E_{sec}}. \end{aligned}\right\} \qquad (2.4)$$

These equations have a lucid physical meaning. Material behaviour can be represented by the two consecutively joined mechanical elements: one, following the crack deformation rule $\sigma \leftrightarrow \xi$, as it is seen in Fig 1, and analytically written as $\sigma = f(\varepsilon^s)$, and another, undergoing linear creep deformations under the action of the stress $\sigma(t)$ according to the equation (2.1). These elements obey the equilibrium condition, the stress σ in both two is equal, and the strain compatibility is also preserved owing to equation (1.3).

Since one of the equations (2.4) is essentially non-linear, solution can be obtained only in an incremental form. If the stress history $\sigma(t'), t_0 \le t' < t$, until the time moment t is known, the stress increment $\sigma(t)$ at the moment can be found from the equations (2.4). Denoting $E_{tan} = d\sigma/d\varepsilon^s$, one can write $d\sigma = E_{tan}\,d\varepsilon^s$, and returning to (2.3), one will obtain

$$d\varepsilon^s = d\varepsilon - d\varepsilon^c + d\sigma/E_{sec}.$$

Next, from the equation (2.1) it will result:

$$d\varepsilon^c(t) = \sigma(t_0)\,dJ(t,t_0) + J(t,t)\,d\sigma(t) + \int_{t_0}^{t} \frac{\partial J(t,t')}{\partial t}\,d\sigma(t').$$

It must be remembered that the variable $\varepsilon^c(t)$ in this equation depends on two arguments: time t and stress $\sigma(t)$. Now for the sake of brevity we introduce appro-priate notations and write the latter equation in the form

$$d\,\varepsilon^{c}(t)=\frac{d\sigma}{E(t)}+d\varepsilon'. \qquad (2.5)$$

Here the first term

$$d\varepsilon'=\sigma(t_0)\,dJ(t,t_0)+\int_{t_0}^{t}\frac{\partial J(t,t')}{\partial t}d\sigma(t') \qquad (2.6)$$

is the deformation increment due to all previous stress history, and the second term represents the deformation increment related to the actual strain increment where

$$\frac{1}{E(t)}=J(t,t).$$

If there are no cracks in the tension zone, we simply have the total strain ε equal to ε^c in equation (2.5). In the presence of cracks we put equations (2.3) and (2.5) together, and obtain main incremental dependence, relating the increment of the total strain ε to the stress increment at the time t:

$$d\varepsilon=d\sigma\left(\frac{1}{E_{tan}}-\frac{1}{E_{sec}}+\frac{1}{E}\right)+d\varepsilon',\quad E_{tan}=\frac{\partial\sigma}{\partial\varepsilon^{s}}\ . \qquad (2.7)$$

The algorithm, presented herein, is intended for use in advanced geometrically and materially non-linear analysis. Even if there are no cracks in the tension zone, and material linearity is preserved, the structure will remain to be non-linear geometrically, and so the incre-mental solution shall be employed. Hence, the incre-mental constitutional relation shall be used instead of direct dependence (2.1) anyhow. In the absence of cracks the linear relation (2.5) will suffice, otherwise relation (2.7) must be used.

3. Incremental stiffness relation for a cross-section

It is assumed that the cross-section is subjected to an axial force F and a bending moment M_y, acting about one of the principal axes of the section. The axes of the reference system not necessarily originate from the centroid of a section, but it is essential that these axes are parallel to the principal axes. Internal forces of the section are resultants of normal stresses:

$$F=\int_A\sigma\,dA,\quad M_y=\int_A z\sigma\,dA\,. \qquad (3.1)$$

Strains are distributed linearly over the section and may be represented as

$$\varepsilon(z)=\varepsilon^{0}+\kappa^{y}z \qquad (3.2)$$

where ε^0 is the strain at the co-ordinate origin point and κ^y is the curvature of a beam axis.

Normal stress σ can be determined using incremental relation (2.6), if creep deformation history is known. Our task is opposite: the section force is given, the deformation must be determined. To solve this problem, an incremental relation between the section deformation parameters and the section forces must be derived.

Let us insert the stress increment $d\sigma=E\left(d\varepsilon^{c}+d\varepsilon'\right)$ from the equation (2.5) into equation (3.1). Incremental relations for section force increments will read

$$\left.\begin{aligned}dF&=J_{00}d\varepsilon^{0}+J_{0y}d\kappa^{y}-dF',\\ dM_y&=J_{y0}d\varepsilon^{0}+J_{yy}d\kappa^{y}-dM'_y.\end{aligned}\right\} \qquad (3.3)$$

The usage of these equations implies a fundamental assumption: the loading of material element must follow the same path in the stress-strain space as its unloading. Otherwise the energy principles will not work, and the symmetry of coefficients, $J_{0y}=J_{y0}$, will not be observed. Neglecting the reloading effects one can substantially simplify the problem, and obtain the coefficients and free terms of (3.3) in the form

$$J_{00}=\int_A E'\,dA,\ J_{0y}=\int_A E'z\,dA,\ J_{yy}=\int_A E'z^2dA\,, \qquad (3.4)$$

$$dF'=\int_A E'd\varepsilon'\,dA,\quad dM'_y=\int_A E'd\varepsilon'z\,dA\,. \qquad (3.5)$$

If there are no cracks, the value E' simply represents the quantity $E(t)$ from equation (2.5). In the cracking stage from the equation (2.7) follows that

$$E'=\frac{d\sigma}{d\varepsilon}=\left(\frac{1}{E_{tan}}-\frac{1}{E_{sec}}+\frac{1}{E}\right)^{-1}. \qquad (3.6)$$

Numerical solution of the problem can be obtained by finite differences, dividing the time period $t_0\le t'\le t$ into appropriate number of sufficiently small intervals. If the stress history is known for $i-1$ previous time intervals, the strain increment during the i-th time interval can be approximately expressed as

$$\Delta\varepsilon_i^{c}=\frac{\Delta\sigma_i}{E_i'}+\Delta\varepsilon'_i\,. \qquad (3.7)$$

Here

$$\frac{1}{E_i'}=\frac{1}{2}\left[J(t_i,t_i)+J(t_{i-1})\right], \qquad (3.8)$$

$$\Delta \varepsilon'_i = \sigma_0 \, \Delta J(t_i, t_0) + \sum_{k=1}^{i-1} \frac{\Delta \sigma_k}{2} \left[\Delta J(t_i, t_{k-1}) + \Delta J(t_i, t_k) \right],$$

$$\Delta\varepsilon_i^c = \varepsilon^c(t_i) - \varepsilon^c(t_{i-1}), \Delta\sigma_i = \sigma(t_i) - \sigma(t_{i-1}),$$

$$\Delta t_i = t_i - t_{i-1}, \Delta J(t_i, t') = J(t_i, t') - J(t_{i-1}, t'). \quad (3.9)$$

These finite difference expressions correspond to the differential expression (2.5) of previous section. Strain increment $\Delta\varepsilon'_i$ in (3.7) is due to stress acting during all previous time intervals. The quantity E_i' here is a certain quasi-elastic modulus, averaged over the i-th time interval, so it is not quite identical with $E(t_i) = 1/J(t_i, t_i)$; the latter strictly corresponds to the time moment t_i.

Rearranging main integral relation (2.4) by means of the above expressions, one will easily obtain finite difference substitute for it:

$$\left.\begin{aligned} &\sigma_i = f(\varepsilon_i^s), \\ &\varepsilon_i^s = \varepsilon_i - \varepsilon_{i-1}^c - \Delta\varepsilon'_i + \frac{\sigma_{i-1}}{E'_i} + \sigma_i\left(\frac{1}{E_{sec}} - \frac{1}{E'_i}\right). \end{aligned}\right\} \quad (3.10)$$

Now the equations (3.3) for section force increments can be rewritten in terms of finite differences.

Substitution of integrals with finite sums causes certain error that can be reduced by the increase of number of time division points. Another source of error is tangent approximation $E_{tan} = d\sigma/d\varepsilon^s$ in equation (2.7). That error is removed by the direct iterative solution of first equation in (3.10). Detailed description of the solution algorithm can be found in [5].

The framework of basic equations, presented in this section, predetermines two levels of solution. At a first level, that is referred to a separate fibre of concrete, the set of two equations (3.8) is solved. The second level comprises the Newton-type solution of incremental equations (3.3), governing the response of the whole reinforced concrete section. Before the occurrence of cracks the problem remains linear, and the solution of equations (3.3) is accurate within the bounds of precision of numerical integration. After the cracking of the tension zone, incrementation along the tangent direction becomes a source of error, and iterative corrections to solutions of equations (3.3) are necessary.

The quantities to be found in equilibrium equations (3.1) are deformation parameters of a section $\varepsilon^0(t_i)$ and $\kappa^y(t_i)$, the given quantities are the section forces $F(t_i)$, $M^y(t_i)$ at the moment, and stress history $\sigma(t_k, z)$, $k = 0, \ldots, i-1$, up to the preceding time moment t_{i-1} for each layer of the section. It must be remembered that relations (3.3) give the force increments that actually take place during the time interval $\Delta t_i = t_i - t_{i-1}$. To construct the algorithm for the above - mentioned iterative corrections one must assume that the section forces in equilibrium equations (3.1) are the functions of deformation parameters only, the time being fixed at the value $t = t_i$. Hence the section force increments in regard of the previously computed values $F(t_i)$, $M^y(t_i)$ will be

$$\left.\begin{aligned} \delta F_i &= J_{00}\,\delta\varepsilon_i^0 + J_{0y}\,\delta\kappa_i^0, \\ \delta M_i^y &= J_{y0}\,\delta\varepsilon_i^0 + J_{yy}\,\delta\kappa_i^0. \end{aligned}\right\} \quad (3.11)$$

The quantities $\delta\varepsilon_i^0, \delta\kappa_i^y$ here must be considered as the corrections to the values $\varepsilon_i^0, \kappa_i^y$ that cause the incre-ments $\delta F_i, \delta M_i^y$ at the unchanged time moment t_i.

4. Computer program

Numerical algorithm was constructed having in mind its inclusion into the major non-linear finite element computer code. The algorithm comprises the Newton-Raphson time-stepping procedure according to equations (3.3) and equilibrium corrections by means of equations (3.11) after every time step. Because of non-linear stress distribution in the tension zone, the numerical evaluation of the sectional integrals in equations (3.1) and (3.3) is employed.

Computational process requires the stress history $\sigma(z, t_k)$, $k = 1, \ldots, i-1$, and the linear creep component $\varepsilon^c(z, t_{i-1})$ at every time step t_i for each concrete layer. In the presence of cracks these quantities cannot be represented as linear functions of co-ordinate z of a layer. Even if the layer is reloaded and crack is closed, the presence of the crack in the past will have been fixed in the stress history, and the linear distribution of layer variables will be destroyed. Nevertheless, it was observed that the distribution of the parameter $\varepsilon^s(z,t)$ over the depth of the section is much more close to the straight line that the stress distribution curve $\sigma(z,t)$. That is important because the large amount of data causes the computer storage problems. Nearly linear disposition of variables enables the condensed storage of data. If the values of ε^s are stored instead of σ quantities, then these quantities are determined from the expression $\sigma = f(\varepsilon^s)$ (1.1), and the creep strains ε^c are obtained by means of equation (2.3).

There is another possible way to reduce the storage space. Material memory of concrete is limited, and so the stress increments at remote time intervals have little influence on its present behaviour. Hence, only most recent stress history must be represented in detail. Former stress increments may be summed and former time intervals may be amalgamated into one interval. That also will appreciably speed up the evaluation of sum $\Delta\varepsilon_i'$ for the equations (3.7). Alternative way to save the storage space is the exponential expansion of creep compliance function. This method is physically motivated by the solidification theory [9]. The creep process is described by the set of the first order linear differential equations. For each time step only the information about preceding step is necessary. The exponential expansion, however, requires sufficiently large number of terms to reflect the stress history until above-mentioned preceding time step. On the other hand, the direct use of creep compliance function is less complicated algorithmically than exponential expansion. Therefore, the creep compliance function and integral representation of analytical model are used here.

The computer program was written for the bending with no axial force. The rectangular reinforced concrete section was assumed. This algorithm can be described in a concise form:

1. Input: time subdivision points t_i, section forces M_i^y, $i = 0, 1, \dots, N$.
2. Set $M_0^y = 0$, $F_i = 0, i = 0,1,\dots,N$, $\varepsilon_0^s = 0$, ε_0^0, κ_0^y.
3. Loop over the time increments: $i = 1, \dots, N$.
4. Compute the section integrals $J_i^{00}, J_i^{0y}, J_i^{yy}$, $\Delta F_i', \Delta M_i^{iy}$ as a finite difference substitute in (3.3).
5. Solve the equations (3.3) for $\Delta\varepsilon_i^0, \Delta\kappa_i^y$ and increment the variables: $\varepsilon_i^0 = \varepsilon_{i-1}^0 + \Delta\varepsilon_i^0$, $\kappa_i^y = \kappa_{i-1}^y + \Delta\kappa_i^y$.
6. Loop over the equilibrium refinement steps: $j = 1, 2, \dots$; set $\varepsilon_{(0)}^0 = \varepsilon_i^0$, $\kappa_{(0)}^y = \kappa_i^y$.
7. Determine the section stress resultants $\tilde{F}_{(j)}, \tilde{M}_{(j)}^y$ from the equations (3.1), and find the unbalanced section forces $\delta F_{(j)} = F_i - \tilde{F}_{(j)}$, $\delta M_{(j)}^y = M_i^y - \tilde{M}_{(j)}^y$.
8. If tolerance condition $\sqrt{(\delta F_{(j)})^2 + (\delta M_{(j)}^y)^2} \le tol$ is satisfied, go to Step 10.
9. Solve the equilibrium refinement equations (3.10), and correct the current values of the variables: $\varepsilon_{(j)}^0 = \varepsilon_{(j-1)}^0 + \delta\varepsilon_{(j)}^0$, $\kappa_{(j)}^y = \kappa_{(j-1)}^y + \delta\kappa_{(j)}^y$.
10. Accept the refined variables $\varepsilon_i^0 = \varepsilon_{(j)}^0$, $\kappa_i^y = \kappa_{(j)}^y$.
11. End of the main loop over the time increments.

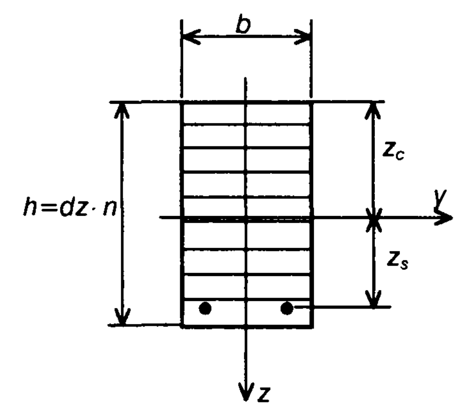

Fig 2. Layer model of a cross-section

To compute the section integrals numerically, the depth of the section was subdivided into n layers as it is shown in Fig 2. Evaluation of section integrals in Step 4 and 7 requires the large amount of intermediate data to be stored. Thus, the economical use of data storage facilities is important. For computation of the section integrals J^{00}, J^{0y}, J^{yy} in absence of cracks simple formula (3.8) is used and no additional information is necessary. If the cracks appear, the equation (3.6) must be used, and the knowledge of the current stress σ_i is required. For computation of the integrals $\Delta F_i', \Delta M_i'^y$ the values of $\Delta \varepsilon_i'$ must be known. These values are computed by means of formulas (3.9). These formulas require the stress history for each layer of concrete irrespective of existence of cracks.

Stress increments are computed directly from equation (3.7), if concrete is uncracked:

$$\Delta \sigma_i = E_i'(\Delta \varepsilon_i - \Delta \varepsilon_i'). \qquad (4.1)$$

Here $\Delta \varepsilon_i$ is the total strain increment. Otherwise, in the cracking stage, stress σ_i must be computed by means of equations (3.10). These equations require the stress at the preceding time step that may be found as $\sigma_{i-1} = f(\varepsilon_{i-1}^s)$, and a linear creep component

$$\varepsilon_{i-1}^c = \varepsilon_{i-1} - \varepsilon_{i-1}^s + \frac{\sigma_{i-1}}{E_{sec}} \qquad (4.2)$$

according to equation (2.3).

Thus, together with the strain history $\varepsilon_k^s, k = 0, \dots, i$, it is desirable to save parameters $\Delta \varepsilon_i'$ for all concrete layers at the latest time step. It is also necessary to save the time of cracking for each concrete layer because it is a turning-point when the stress reaches its maximum value $\sigma = f_t'$ and then begins to drop.

The computation of section integrals is separated into the special subroutine. The algorithm is described in [6].

5. Constitutional relations of concrete

Until the occurrence of cracks, concrete follows the linear creep law. After cracking the behaviour of concrete is governed by the interaction of concrete creep and strain-softening mechanisms.

Any analytical model for concrete creep may be employed in the FORTRAN program, described herein. All modern models comprise more or less complicated analytical expressions $J(t,t')$ for creep compliance, that depends on the age of concrete and is the function of a current time t and the time t' when stress increment was applied. The creep compliance function [8], which is used herein, was chosen because of a number of advantages: (1) It is well-based physically and attributes the aging properties of concrete to solidification process of concrete paste [9]. (2) The solidification theory facilitates conversion to a rate-form of the constitutive relation, the latter being more preferable for step-by-step numerical procedures. (3) The drying effects are separated from the basic creep process which takes place at no drying and no temperature change; these two components of creep are of different physical nature and originate from different physical mechanisms. (4) This mathematical model can be easily fitted to existing experimental data because material parameters appear in a linear form, and linear regression can be used.

The basic principle of the solidification theory is that the aging of concrete is determined by a growth of amount of hydrated cement, which itself can be described as non-aging material. Long-time strain of concrete is seen as the sum

$$\varepsilon = \frac{\sigma}{E_0} + \varepsilon^v + \varepsilon^f + \varepsilon^0, \qquad (5.1)$$

where σ — normal stress, ε^v — viscoelastic creep strain, ε^f — flow strain, ε^0 — shrinkage strain, including stress-induced shrinkage. The elastic modulus E_0 here is an asymptotic constant quantity, representing deformation due to extremely short load durations. Conventional elastic deformation according this theory is only apparent phenomenon, and in reality must by treated as a short-time creep. We shall consider in this article only the three former components of the strain, omitting the shrinkage.

The derivation of the creep compliance rate [9] is not repeated here. The analytical expression for basic creep is rather simple and reads as follows:

$$\dot{J}(t,t') = \left(\frac{q_2}{t^m} + q_3\right) \frac{n(t-t')^{n-1}}{1+(t-t')^n} + \frac{q_4}{t}. \qquad (5.2)$$

Here $m = 0.5$, $n = 0.1$ are empirical constants. The quantities q_1, q_2, q_3, q_4 are material parameters: $q_1 = 1/E_0$ represents the pure elastic strain, q_2, q_3, q_4 correspond to the aging viscoelastic compliance, the non-aging viscoelastic compliance and the flow compliance, respectively. Integrating with the initial condition $J(t,t) = q_1 = 1/E_0$, one can obtain

$$J(t,t') = q_1 + q_2\, Q(t,t') + q_3 \ln(1+t-t') + q_4 \ln(t/t') \qquad (5.3)$$

where $Q(t,t')$ is a term that cannot be expressed analytically. In [9] approximate formulas are proposed for this quantity.

Material parameters $q_1,\dots,q_4$ must be adjusted to the available experimental results. The analytical creep curves are sufficiently smooth for a broad range of load durations from parts of a second up to 30 years, and even the short-time experimental results are useful. In the absence of test data, these parameters may be estimated according to concrete strength and mix composition [8].

The conventional elastic modulus of concrete cor-responds to loading duration approximately $\Delta t = 0.1$ day and can be determined from the compliance function as

$$E(t) = 1/J(t+\Delta t, t). \quad (5.5)$$

Thus the asymptotic quantity q_1 may be equated to a 28-day modulus E_{28} using equation

$$E_{28} = J(28.1, 28.0). \quad (5.6)$$

According to [9]

$$E_{28} = 4733.\sqrt{f_c}. \quad (5.7)$$

The creep properties of concrete can be described clearly by the creep coefficient

$$\varphi(t,t') = E(t')\, J(t,t') - 1. \quad (5.8)$$

The value $\varphi(t,t')$, however, may be misleading because the elastic modulus $E(t')$ is determined on the basis of somewhat arbitrarily chosen time interval Δt.

In sample calculation following concrete mix parameters were chosen: C = 320.0 kg/m^3, W/C = 0.5, a/C = 0.7. Only basic creep was treated, drying creep was neglected. The 28-day cylinder strength of concrete was $f_c' = 30.0$ MPa. Material parameters were computed according to [9]:
$q_1 = 3.184\cdot10^{-5}$, $q_2 = 5.933\cdot10^{-5}$, $q_3 = 1.483\cdot10^{-6}$, $q_4 = 1.214\cdot10^{-5}$, MPa^{-1}. Conventional 28-day elastic modulus $E_{28} = 25920$ MPa, asymptotic value of initial instantaneous elastic modulus $E_0 = 31410$ MPa. In Fig 3 the growth of elastic modulus $E(t)$ is shown. Here $E(t_0) = 23230$ MPa, $t_0 = 10.0$ days; elastic modulus by the end of a lifetime, t = 10000 days is $E(t) = 30930$ MPa. In Fig 4 computed creep coefficient $\varphi(t,t_0)$ with time of

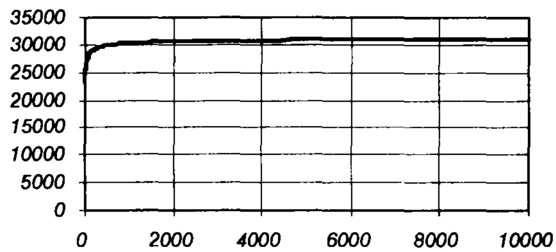

Fig 3. Elastic modulus $E(t)$, computed on the basis of eq. (5.5)

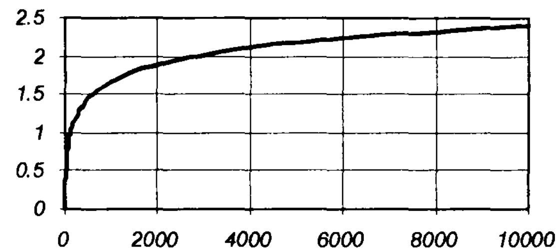

Fig 4. Creep coefficient $\varphi(t, t_0)$ according to eq. (5.8)

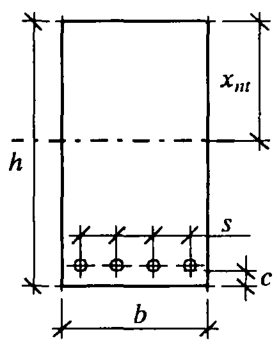

Fig 5. Cross-section of a sample member h = 0.50 m, b = 0.25 m, c = 0.04 m, s = 0.05 m, diameter of a reinforcement bar 20 mm

loading $t_0 = 10.0$ days is presented. The final value of the creep coefficient is $\varphi(t,t_0) = 2.396, t = 10000$ days.

Tension behaviour of concrete depends on the parameters of a cross-section. Strain-softening parameter β of concrete in tension was computed according to equation (1.2) with numerical parameters of the section that are shown in Fig 5. Since the depth $d - x_{nt}$ of the tension zone has rather small influence on the value of β, it was taken approximately $x_{nt} = 0.5d$. Cracking strain was assumed $\varepsilon_t' = 0.0002$. The tensile strength of concrete was determined by the empirical formula [10]:

$$f_t' = 0.324\sqrt[3]{f_c^2}\,, \tag{5.9}$$

$f_t' = 3.128$ MPa. The increase of concrete strength during the lifetime of the member was neglected for the sake of simplicity. The elastic modulus of steel is taken $E_s = 200\,000$ MPa, steel yielding stress is $f_y = 320$ MPa.

6. Long-time behaviour of a section

The cross-section of the reinforced concrete member was subjected to sustained bending moment with a zero axial force. Nearly constant regime of loading was simulated. It was found algorithmically more convenient to apply steadily growing bending moment

$$M(t) = M_1 \frac{\ln(t/t_0)}{\ln(t_1/t_0)} \tag{6.1}$$

at the short initial period of time, $t_0 \le t \le t_1$, and then keep the constant value of M_1 in the remaining time period $t_1 < t \le t_N$. The entire period is subdivided into a N time intervals. Time subdivision points $t_1, \ldots, t_N$ are assumed according to a geometrical progression. It also was assumed $t_1 = 20.0$ days, $t_N = 10000.0$ days.

The rectangular cross-section was initially divided into $n = 50$ layers. To evaluate the numerical error, calculation were repeated with doubled number of layers. The number of time steps over the lifespan of the member was alternated from 200 to 10. The check of accuracy revealed that subdivision into $n = 50$ layers and 50 time steps brings the difference of steel strain 0.17 % by the end of the lifetime in comparison with maximum number of steps 200 and number of layers $n = 100$. The difference in concrete stresses is even smaller. 20 and 10 time steps bring 0.66 % and 1.36 % error, respectively. So one can draw the conclusion that 50 layers of the section and 50 time steps in the case of a constant sustained load give an acceptable precision of numerical results.

An attempt was also made to reduce the necessary computer storage space by cutting off the most remote points of the stress history. Retaining only 10 latest of the entire 50 stress history points, the 0.66 % error in the steel stress at the last time moment has been observed. Representation of the stress history by the 25 latest time points results in the 0.21 % error of steel stress.

Numerical results are briefly reviewed in Fig 6 and Fig 7, where diagrams of normal stresses σ and equivalent instantaneous strains ε^s are presented. The diagrams are plotted for the time moment $t_1 = 20.0$ days when the bending moment reached its maximum value M_1 and for the final time moment $t_N = 10000$ days, at the time steps # 10 and # 100, respectively. It is obvious that stress diagram in the cracked zone is essentially non-linear, but the equivalent instantaneous strain ε^s varies over the cracked zone in almost linear manner. Linear distribution simplifies representation of this quantity in the tension zone. This circumstance is of significant importance because of large demands for storage space in computer program. It is also seen that significant stress redistribution in the section due to sustained load action occurs.

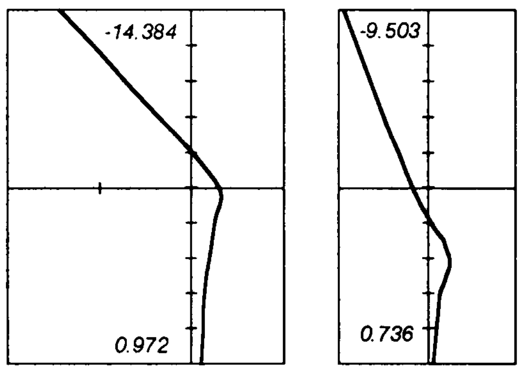

Fig 6. Stress distribution over the cross-section at the time moments t_1 = 20 and t_N = 10000 days

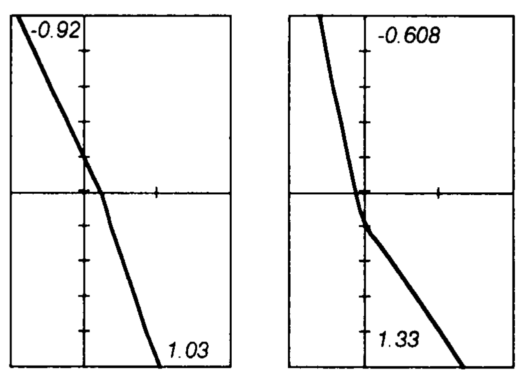

Fig 7. Equivalent instantaneous strains $\varepsilon^s \times 10^3$ at the time moments t_1 = 20 and t_N = 10000 days

In Fig 8 redistribution of internal forces between the concrete and tension reinforcement is shown. One can see that the compressive stress of concrete is falling, and tensile stress of reinforcement steel is increasing.

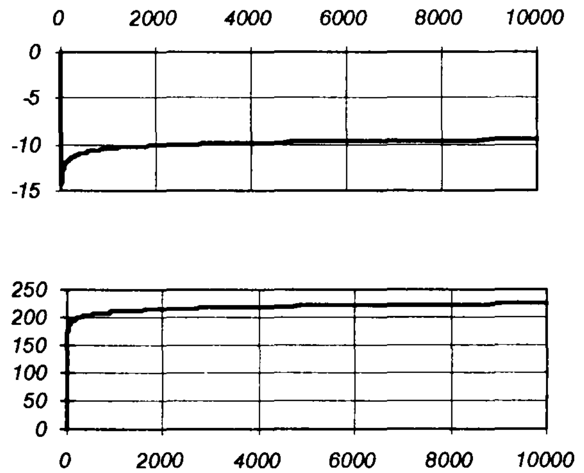

Fig 8. Compressive stress in concrete (upper graph) and stress in tension reinforcement (MPa) as a functons of time (days)

7. Summary and conclusions

Incremental stiffness algorithm for reinforced concrete cross-section creep analysis is developed. The layer model for plane bending is employed. Strain-softening behaviour of concrete in tension is modelled, assuming the additivity of linear creep and cracking strain. Further research is needed to investigate the influence of reloading effects. That will be the subject of subsequent publication.

In the proposed algorithm linear time-stepping procedure for concrete creep is combined with step-by-step incrementation of non-linear cracking strains. Two possibilities were also investigated and proposed to reduce the data storage requirements. (1) Condensed representation of stress history, using only the latest stress increments, and summing the rest of earlier stress increments into one quantity. (2) Representing the stress in the section by equivalent strain ε^s as an argument of non-linear stress function (2.3) in the cracking stage; in this way nearly linear distribution of the quantity over the tension zone is utilized to avoid the separate storage of data for each concrete layer.

Within the scope of this study the following conclusions can be drawn:

1. The non-linear stepwise section stiffness algorithm for creep analysis has shown good convergence properties.
2. An error of stress not exceeding 0.2 % can be achieved in the case of sustained load, dividing the cross-section into 50 layers or more, and employing not less than 50 time steps.
3. The computation can be speeded up and data storage space significantly saved, using condensed representation of stress increments, remote from the current time moment. The use of equivalent strain to represent the non-linear tensile stress in concrete also can reduce the amount of stress data.

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LENKIAMŲ GELŽBETONINIŲ ELEMENTŲ SKAITMENINĖ VALKŠNUMO ANALIZĖ

I. Cypinas

Santrauka

Betono pleišėjimo analitinis modelis esant valkšnumui, pateiktas autoriaus jo publikacijoje [5], yra taikomas lenkiamų gelžbetoninių elementų valkšnumo analizei. Nagrinėjamas plokščias elemento lenkimas esant vienai simetrijos ašiai. Skaičiavimas atliekamas baigtiniais laiko intervalais, taikant analitinę betono valkšnumo funkcijos išraišką (5.3).

Gniuždymo zonos betonui taikoma tiesinio valkšnumo teorija (2.1). Tempiamai betono zonai taikoma netiesinė deformacijų prieaugių procedūra. Skerspjūvio standumo parametrai randami, taikant skaitmeninį integravimą. Skaičiavimo algoritmas yra pagrįstas Niutono metodu netiesinėms skerspjūvio deformacijos prieaugių lygtims (3.3) spręsti, kiekvienam laiko intervalui nustatant liestines skerspjūvio standumo charakteristikas.

Pagrindinis darbo tikslas yra sukurti efektyvų algoritmą gelžbetoninio skerspjūvio valkšnumo deformacijų analizei ir ištirti galimybes taikyti šį valkšnumo analizės metodą didelių sistemų skaičiavimui baigtiniais elementais. Pagal sukurtą algoritmą autorius sudarė FORTRANO programą skerspjūvio deformacijoms skaičiuoti. Siekiant išaiškinti algoritmo galimybes, buvo atliktas valkšnumo deformacijų skaičiavimas veikiant pastoviam lenkimo momentui laikotarpiu nuo 20 iki 10 000 parų, skaitant nuo betono stingimo pradžios. Betono valkšnumas buvo analitiškai aprašomas pagal darbų ciklo [8] rekomendacijas. Nustatyta, kad sudalinant skerspjūvį į 50 skaitmeninio integravimo sluoksnių ir dalinant visą apkrovimo laikotarpį į 50 laiko intervalų yra gaunama 0.17% armatūros įtempimų paklaida paskutiniame laiko žingsnyje, palyginus su baziniu skaičiavimu su 100 integravimo sluoksnių ir 200 laiko intervalų.

Deformacija kiekviename laiko intervale priklauso ne tik nuo įtempimų tame intervale, bet ir nuo visos įtempimų istorijos. Tačiau reikšmingą įtaką turi tik įtempimų prieaugiai pastaraisiais laikotarpiais, ankstesnių laiko intervalų įtempimų prieaugiai gali būti susumuoti į vieną dydį, taip sumažinant laikomos informacijos apimtį. Skaičiavimais nustatyta, kad iš 50 laiko žingsnių palikus tik paskutiniųjų 10 informaciją apie įtempimų prieaugius, gaunama 0.21% armatūros įtempimų paklaida. Be to, šitoks įtempimų istorijos supaprastinimas gerokai pagreitina skaičiavimo procesą, kadangi sumažėja (3.9) formulės sumuojamų dėmenų skaičius.

Įtempimai yra vaizduojami netiesiogiai, naudojant ekvivalentę deformaciją (2.3) kaip netiesinės funkcijos įtempimams rasti argumentą. Ši deformacija pasiskirsto supleišėjusios zonos aukštyje tiesiškai, tuo tarpu kai įtempimų diagrama yra ryškiai kreivalinijinė. Ši aplinkybė įgalina daug taupiau vaizduoti tempimo zonos įtempius.

Sudarytoji programa ir ja atlikti skaičiavimai rodo, kad siūlomas algoritmas gali būti sėkmingai pritaikytas didesnių konstrukcinių sistemų analizei baigtinių elementų metodu.

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