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1. Introduction

The creep of concrete in compression is adequately described by the integral-type stress-strain relation according to ageing linear viscoelasticity. The linear creep law can be used for a compression zone, because the long-time loading of structures rarely exceeds a half of a total load. The failure of the structure is usually caused by a sudden overloading, and creep phenomenon is not involved in the failure stage. Linear creep law can also be applied to concrete in tension. However, cracks occur in the tension zone of reinforced concrete members in early stages of deformation, and the influence of cracks must be accounted for when evaluating the deformation of the tension zone.

There are two different approaches to this issue. First, more traditional one, is based on the assumption that in the tension zone suddenly occur sharp continuous cracks, but intact concrete blocks between the cracks work together with the tensile reinforcement. The contribution of the concrete in tensile zone depends on the bond stress distribution along the length of the uncracked concrete block. This phenomenon is known as tension-stiffening, and there are many publications on this problem. In Ref. [1] an experimentally based bilinear bond stress-slip curve is used to obtain an analytical solution for cracking behaviour of a reinforced concrete member. Ref. [2] contains analytical solution based on certain non-linear bond-slip relations [3]. In Ref [4] an a priori bond stress distribution is assumed. That approach was widely known in Russia from early fifties [5], and later it became very popular in other countries too.

Slightly different approach is used in Eurocode 2 [6]. The concrete segment between the cracks is divided into two regions: region I, where concrete interacts with the reinforcement fully, and region II, close to the crack, where bond is damaged and concrete does not interact with the reinforcement at all. The relative length of these two regions is defined by the parameter

$$\zeta = 1 - \beta_1 \beta_2 \left(\sigma_{sr} / \sigma_s\right)^2, \qquad (1.1)$$

where σ_{sr} is steel stress at cracking, σ_s — steel stress under the actual loading, the both stresses being evaluated on the basis of a cracked section. Empirical coefficient β_1 characterizes the bond properties of reinforcement bars, $\beta_1 = 0.5$ for plain bars, $\beta_1 = 1.0$ for ribbed bars, and β_2 is a coefficient taking account of load duration. For a short-term loading $\beta_2 = 1.0$, for a long-term or repeated loading $\beta_2 = 0.5$. Theoretical justification for this approach can be found in Ref. [7].

In all these sources concrete in tension as well as in compression is treated as linearly elastic. But concrete in tension does not behave as ideally elastic brittle material. Thoroughly conducted experiments pointed out that tension test specimens exhibit long descending branch of their stress–strain curve before they break. That is because of microcracks, occurring in early stages of tension and gradually coalescing into the major cracks when stress approaches the tensile strength. This phenomenon is known as strain-softening. Its analytical description for the short–time loading is presented in Ref. [8]:

$$\sigma = f_t' \frac{\beta x}{\beta - 1 + x^{\beta}}, \quad x = \frac{\varepsilon}{\varepsilon_t'} \qquad (1.2)$$

where f_t' is tensile strength of concrete, ε_t' — the strain at f_t', β — an empirical parameter.

It is known that the deformation process with the falling stress–strain curve is unstable [9]. The tension reinforcement and adjacent concrete in compression zone of a beam stabilizes the tensile deformation of concrete. The stabilizing effect depends on the character of reinforcement and the depth of the tension zone. These factors determine the magnitude of β and σ in (1.2). In Ref.

[10] a slightly simplified stress-strain curve is presented for practical use :

$$\sigma = \begin{cases} f_t' x, & x = \dfrac{\varepsilon}{\varepsilon_t'}, \quad \text{if} \quad x \le 1, \\ f_t' \dfrac{\beta x}{\beta - 1 + x^{\beta}}, & \text{if} \quad x > 1. \end{cases} \tag{1.3}$$

The value of β, adjusted to the experimental results, is expressed as

$$\beta = \left(\frac{100 A_s}{b(h - x_{nt})} \right)^{0.366} \left(\frac{b(h - x_{nt})}{n \pi c d} \right)^{0.3436} \left(\frac{c}{s} \right)^{0.146}, \tag{1.4}$$

where A_s — area of tensile reinforcement, b, h — width and depth of the rectangular cross–section, n — number of reinforcing bars, d — diameter of the reinforcing bars, c — concrete cover to reinforcement, s — reinforcement spacing; x_{nt} is the neutral axis depth, computed neglecting tension in concrete. It must be noted that there is a rather weak dependence between the β and x_{nt}, and so the value of x_{nt} may be taken approximately.

For the descending branch of a stress-strain curve may also be used a simplified linear representation, as it is done in Refs. [11], [12]:

$$\sigma = \begin{cases} E_c \varepsilon, & \text{if } \varepsilon \le \varepsilon_{tp}, \\ f_t' + (\varepsilon - \varepsilon_{tp}) E_t, & \text{if } \varepsilon_{tp} < \varepsilon \le \varepsilon_{tf}, \\ 0, & \text{if } \varepsilon > \varepsilon_{tf}. \end{cases} \tag{1.5}$$

Here E_c — initial modulus of concrete, $\varepsilon_{tp} = f_t'/E_c$, ε_{tf} — ultimate strain when the tensile stress is reduced to zero. In this formula strain-softening modulus E_t is

$$E_t = \frac{0.483 E_c}{0.393 + f_c'}. \tag{1.6}$$

As to the creep deformations in the presence of cracks, there are rather few publications on this issue. In the monograph [13] an usual tension-stiffening approach is employed to the creep phenomenon. Z. P. Bažant and co-authors in Refs. [11], [12] substantiates the strain-softening approach. Experimental evidence shows that strain-softening is a primary mechanism of concrete-reinforcement interaction in the tension zone. In Ref. [14] author applies this concept to tension creep deformation in the cracking stage. Deformations due to cracking and linear creep deformations are treated as additive quantities. Substituting creep strains for the elastic component of strain in constitutive relation (1.3), one obtains the strain-softening relation in the case of creep. In Ref. [15] strain-softening concept is also applied to creep deformation of structures.

2. Incremental Form of Creep Relations for Concrete

The creep of concrete in compression is described by the linear integral relation

$$\varepsilon(t) = \sigma(t_0) J(t, t_0) + \int_{t_0^+}^{t} J(t, t') d\sigma(t'). \tag{2.1}$$

Here $J(t,t')$ is a creep compliance function. Superscript $^+$ over the lower limit of integration means that the point $t=t_0$ does not include in the integration interval.

When solving non-linear deformation problems one must formulate them in an incremental form. In the case of creep deformation that can be achieved in a natural way by using a finite difference representation of integro-differential equation (2.1) under the condition that the stress history of the structure $\sigma = \sigma(t)$ is represented by the broken line:

$$\begin{aligned} \Delta \varepsilon_i = \sigma_0 \Delta J(t_i, t_0) + \frac{\Delta \sigma_i}{\Delta t_i} \int_{t_{i-1}^+}^{t_i} J(t_i, t') dt' + \\ + \sum_{k=1}^{i-1} \frac{\Delta \sigma_k}{\Delta t_k} \int_{t_{k-1}^+}^{t_k} \Delta J(t_i, t') dt', \end{aligned} \tag{2.2}$$

$$\Delta t_i = t_i - t_{i-1}, \quad \Delta \varepsilon_i = \varepsilon(t_i) - \varepsilon(t_{i-1}),$$
$$\Delta \sigma_i = \sigma(t_i) - \sigma(t_{i-1}),$$
$$\Delta J(t_i, t') = J(t_i, t') - J(t_{i-1}, t').$$

Eq. (2.2) can be regarded as a quasi–elastic relation

$$\Delta \varepsilon_i = \frac{\Delta \sigma_i}{E_i'} + \Delta \varepsilon_i', \quad i = 1, \ldots, N, \tag{2.3}$$

where the first term represents strain increment due to $\Delta \sigma_i$,

$$\frac{1}{E_i'} = \frac{1}{\Delta t_i} \int_{t_{i-1}^+}^{t_i} J(t_i, t') dt', \tag{2.4}$$

and the second one is deformation due to all previous stress increments:

$$\Delta \varepsilon_i' = \sigma_0 \Delta J(t_i, t_0) + \sum_{k=1}^{i-1} \frac{\Delta \sigma_k}{\Delta t_k} \int_{t_{k-1}^+}^{t_k} \Delta J(t_i, t') dt'. \tag{2.5}$$

Eq. (2.3) is exact under the condition that the strain history is in the form of the broken line. That formula necessitates the storing of all stress history, but this task is simplified when stress is distributed linearly over the cross–section of the member.

When a problem is formulated in terms of displacements, the explicit representation of stresses is required, and principal constitutive relation (2.1) ought to be inverted. That can be achieved by use of a relaxation function $R(t,t')$. In that case the constitutive relation reads as

$$\sigma(t) = \varepsilon(t_0) R(t,t_0) + \int_{t_0^+}^{t} R(t,t')\, d\varepsilon(t') \qquad (2.6)$$

and incremental form of this relation is

$$\Delta\sigma_i = \Delta\varepsilon_i E_i' + \Delta\sigma_i' . \qquad (2.7)$$

Here the first term is the stress increment due to the strain increment at the last time interval, and the second one is due to all previous strain increments.

Incremental relation Eq. (2.3), as well as Eq. (2.7), can be inverted; both relations are equivalent. The strain history of member in flexure can easily be stored owing to the linear distribution of strains over the cross–section. The first formulation, however, is more preferable, because the creep compliance function $J(t,t')$ is formulated analytically, while the relaxation function $R(t,t')$ must be computed by the numerical integration.

3. Analytical model for creep of tension zone in the cracking stage

As it was mentioned above, numerical modelling of the reinforced concrete tension zone can be accomplished either by use of the strain-softening concept or traditional stress-stiffening approach. Here we shall generalize the former concept to the case of creep deformation, because this concept is more convenient computationally.

Time-dependent tensile deformation of concrete is governed by the linear creep law until the tensile strength is reached and tensile cracks occur. Then the total strain is expressed as a sum of linear creep deformation ε^c of concrete between the cracks, and cracking deformation ξ due to the opening of these cracks:

$$\varepsilon = \varepsilon^c + \xi . \qquad (3.1)$$

Last component in this equation represents the strain-softening effect, and it is assumed to be the time-independent, such as in the case of short-time loading. Analytical model is constructed analogically as in Ref. [14]. The model can be seen as two elements, coupled in series. First of them is a linear creep element, undergoing *long-time* deformation, and another is the element, undergoing deformation ξ, that is the inelastic component of the *short-time* strain after the occurrence of cracks.

Linear creep component of the total strain causes the stress, determined by the relaxation law:

$$\sigma(t) = \varepsilon^c(t_0) R(t,t') + \int_{t_0^+}^{t} R(t,t')\, d\varepsilon^c(t') . \qquad (3.2)$$

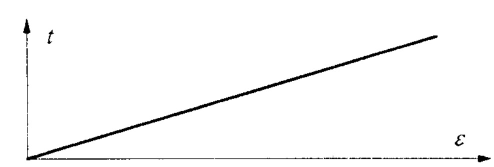
This equation results in the incremental relation (2.3). The stress $\sigma(t)$ must also satisfy the strain softening law according Eq. (1.3):

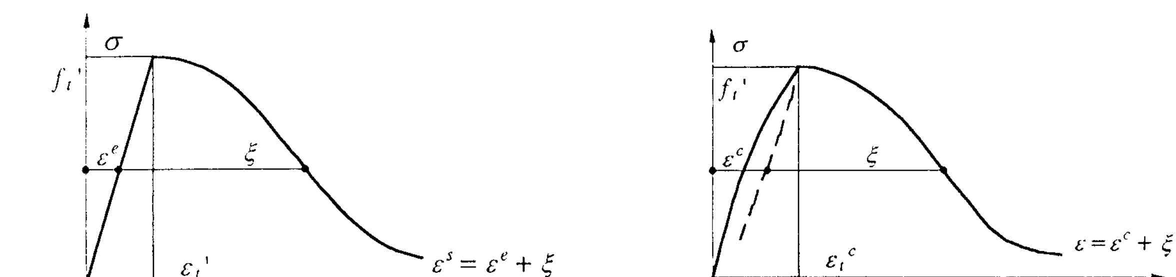
$$\sigma = f_t' \frac{\beta x}{\beta - 1 + x^{\beta}}, \quad x = \frac{\varepsilon^s}{\varepsilon_t'} . \qquad (3.3)$$

Here ε^s is a certain imaginary short–time strain, $\varepsilon^s = \varepsilon^e + \xi$, containing the same value of ξ, as long–time strain in Eq. (3.1) (see Fig. 3.1 a). Combining the last equality with Eq. (3.1), we obtain the following expression, relating ε^s to the imposed total strain ε:

$$\varepsilon^s = \varepsilon - \varepsilon^c + \varepsilon^e \qquad (3.4)$$

b)





a)

Fig. 3.1. Cracking behaviour of concrete: a) cracking without time effect, b) coupling of creep and cracking deformations.

Elastic strain ε^e, as a component of ε^s is seen in Fig. 3.1 b. It equals

$$\varepsilon^e = \frac{\sigma}{E_c}, \quad E_c = \frac{f_t'}{\varepsilon_t'}. \tag{3.5}$$

Eqs. (3.2) to (3.5) determine the time-dependent deformations of concrete after the occurrence of cracks. First of this set is integro-differential equation, hence, the numerical solution must be of incremental form. Rearranging Eq. (3.4) to an incremental form, and inserting into it constitutive relations in the form of Eqs. (2.3) and (3.5), one obtains:

$$\Delta\varepsilon_i^s = \Delta\varepsilon_i + \frac{\Delta\sigma_i}{E_c} - \frac{\Delta\sigma_i}{E_i'} - \Delta\varepsilon_i' \tag{3.6}$$

Further, denoting expression Eq. (3.3) as $\sigma = \sigma(\varepsilon^s)$, and its derivative as $E^{tg} = \dfrac{d\sigma}{d\varepsilon^s}$, one will obtain approximate equality $\Delta\varepsilon_i^s = \dfrac{\Delta\sigma_i}{E_i^{tg}}$. In the sequel, incremental tangent relation for $\Delta\sigma_i$ will be derived from Eq. (3.6):

$$\Delta\sigma_i\left(\frac{1}{E_i^{tg}} - \frac{1}{E_c} + \frac{1}{E_i'}\right) = \Delta\varepsilon_i - \Delta\varepsilon_i' \tag{3.7}$$

It should be noted that in reality the quantity in brackets at $\Delta\sigma_i$ is negative; hence, σ_i must be positive, if $\Delta\varepsilon_i = 0$ in the right-hand-side of the Eq. (3.7). Positive increment of stress with the constant strain in the range of falling stress-strain curve is physically unjustifiable. To avoid that discrepancy an appropriate reloading law must be introduced for cracking strain ξ. We shall circumvent that difficulty excluding the case of constant strain from the consideration and imposing a small strain increments $\Delta\varepsilon_i > \Delta\varepsilon_i'$ to ensure the negative stress increments in the Eq (3.7), and non-negative increments of ξ.

The solution σ_i at the end of the i–th time step must satisfy the relation

$$\varepsilon_i^s = \varepsilon_i - \varepsilon_{i-1}^c - \Delta\varepsilon_i' + \frac{\sigma_{i-1}}{E_i'} + \sigma_i\left(\frac{1}{E_c} - \frac{1}{E_i'}\right), \tag{3.8}$$

which is obtained from the strain compatibility equation (3.4). Substituting σ_i, as a function of ε_i^s into the right-hand-side of the last equality, we arrive at the equation for the unknown quantity ε_i^s. Since the relation between the σ_i and ε_i^s is non-linear, the incremental solution according the Eq. (3.7) must be refined after every time step. The refinement may be performed by direct iteration, using a recurrent expression

$$\varepsilon_{(j+1)}^s = \varepsilon_i'' + \frac{\sigma(\varepsilon_{(j)}^s)}{E_i''}, \tag{3.9}$$

where

$$\varepsilon_i'' = \varepsilon_i - \varepsilon_{i-1}^c - \Delta\varepsilon_i', \quad \frac{1}{E_i''} = \frac{1}{E_c} - \frac{1}{E_i'}. \tag{3.10}$$

The iteration process is shown in Fig. 3.2. In the case of linear falling branch of the stress-strain relation (1.5) above described iterative refinement will become unnecessary.

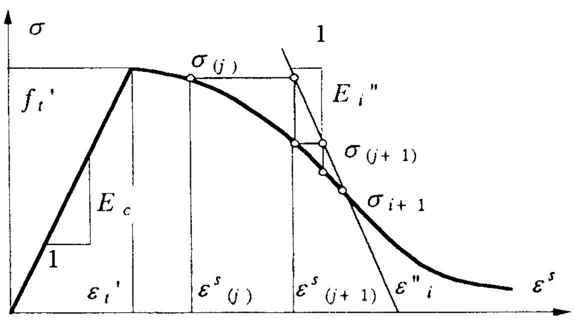


Fig. 3.2. The iterative solution of Eq. (3.9).

4. Computer implementation and numerical results

In order to investigate the behaviour of the proposed creep model, the FORTRAN computer program was written. It was important to ascertain the influence of the creep strain component on the cracking process.

Until the cracks appear, the linear incrementation scheme is valid. After the tensile stress reaches its maximum value f_t', the cracking strain component ξ appears, and the falling branch of the stress-strain relation comes into effect. The computer program contains two loops, enclosed one into another, the outer loop performing the time steps over the lifetime of the structure, and the inner one performing the iterative refinement of the incremental solution $\sigma(t_i)$ according Eq. (3.9) for the time moments $t_i, i = 1, \ldots, N$.

The algorithm is presented below.

1. Beginning of the strain incrementation loop. Set $i = 1$.
2. Calculate E_i' from Eq. (2.4), and $\Delta\varepsilon_i'$ according to Eq. (2.3).
3. Calculate the stress increment $\Delta\sigma_i$ from Eq. (2.3).

4. If $\sigma_i < f'_t$ then assume $i = i + 1$ and go back to Step 2, else assume $i_\xi = i$.
5. Determine by linear interpolation the time $t = t_\xi$, when the stress $\sigma_\xi = f'_t$ is reached.
6. Beginning of the cracking stage. Assume $i = i_\xi + 1$.
7. Calculate E''_i from Eq. (3.10).
8. If $i = i_\xi + 1$ then go to Step 11.
9. Calculate $E_i^{tg} = d\sigma / d\varepsilon^s$ from Eq. (1.3).
10. Calculate $\Delta\sigma_i$ from Eq. (3.7).
11. Calculate ε''_i according to Eq. (3.10).
12. Beginning of equilibrium iterations. Set $j = 1$.
13. Calculate ε^s_{j+1} from Eq. (3.9).
14. Calculate σ_{j+1} from Eq. (1.3).
15. If convergence is not attained, then go back to Step 13.
16. If ultimate strain is not reached, then go back to Step 7.

A separate layer of the reinforced concrete beam was analysed, using that algorithm. The cross-section of the beam is shown in Fig. 4.1.

$h = 0.50$ m
$b = 0.25$ m
$c = 0.04$ m
$d = 0.013$ m
$s = 0.05$ m

$x_{nt} \approx 0.5h$

$A_s = n\dfrac{\pi d^2}{4}, n = 4$

d — diameter of reinforcement bars

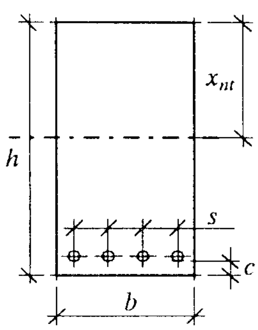


Fig. 4.1. The cross-section and parameters of the beam

The creep parameters of concrete were evaluated according to the analytical prediction model Ref. [16]. The 28-day cylinder strength of concrete was assumed $f_c' = 27.58$ MPa. Concrete mix composition parameters were as follows:

$C = 320.4$ kg/m^3 — specific contents of cement,

$W/C = 0.5$ — water/cement ratio by weight,

$a/C = 0.71$ — aggregate/cement ratio by weight.

The tensile strength of concrete was determined by a formula

$$f_t' = 0.324\sqrt[3]{f_c'^2}. \tag{4.1}$$

The model was subjected to forced deformation process. Several different regimes of deformation were selected, as it is shown in Fig. 4.2. The regimes are characterised by the ratio

$$\omega = \frac{\int_{t_0}^{t_N} \varepsilon(t)dt}{\varepsilon(t_N)(t_N - t_0)}. \tag{4.2}$$

The concrete strength increase during the time interval is neglected for the sake of simplicity. The first form of the process represents the case of almost constant strain, $\omega = 0.95$. Strictly constant regime was excluded to avoid the reversal of the crack strain component ξ. The second strain-versus-time curve is the linear growth of strain, $\omega = 0.5$, the third curve is the sudden growth of the strain near the end t_N of the lifetime, $\omega = 0.07625$.

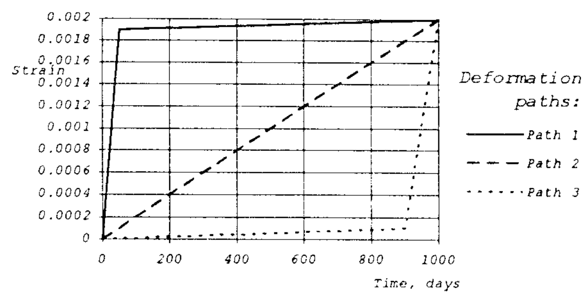


Fig. 4.2. Characteristic strain histories, adopted in example calculations

The stress history was computed by means of the above mentioned computer program for all strain regimes. Results are presented in the Fig. 4.3. It is seen that the stress differences at an ultimate state due to different values of ω are rather small; the stress depends mainly on the strain level at the moment.

5. Summary and conclusions

Numerical investigation of the concrete creep model in the presence of cracks have been carried out. The incremental form of creep relations, based on the creep compliance function, was used throughout. Computer code was developed to evaluate the tensile stresses of cracked concrete, subjected to the long-time axial strain, growing according to a given law.

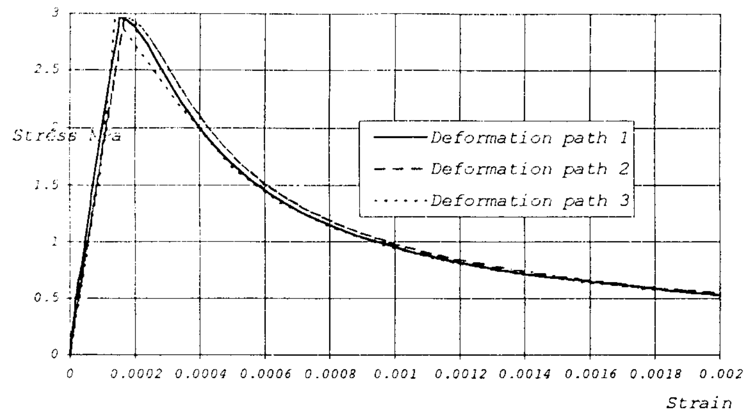


Fig. 4.3. Stress histories for different deformation paths

The following conclusions may be drawn from this study:

1. The non-linear series model for concrete creep in the presence of tensile cracks have shown good convergence.
2. The creep has rather small influence on the cracking and strain-softening behaviour of concrete in tension.
3. Simplified model of the cracking, based on the linear approximation of the creep component of strain, can be suggested.

The numerical procedures, presented herein, are intended to introduce into the finite element computer code for creep analysis of reinforced concrete beams and frames. That will be the subject of subsequent publications.

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VALKŠNUMO SKAITMENINIS MODELIAVIMAS GELŽBETONINIO ELEMENTO TEMPIMO ZONOJE

I. Cypinas

Santrauka

Aprašant tempiamo betono valkšnumą, būtina atminti, kad jau ankstyvose darbo stadijose betone atsiranda plyšiai. Yra žinomi du analitiniai modeliai, aprašantys tempiamos armatūros ir pleišėjančio betono sąveiką. Pirmasis, labiau tradicinis modelis postuluoja staigiai atsirandančius ištisinius plyšius, tarp kurių lieka nesupleišėję betono blokai, dalyvaujantys tempiamos zonos darbe dėka armatūros ir betono sukibimo. Augant deformacijoms toliau, palaipsniui nyksta armatūros sukibimas su betonu ir tempiamas betonas palaipsniui išsijungia iš darbo. Kitame modelyje pleišėjantis betonas traktuojamas kaip ištisinis kūnas, o tarp betono ir armatūros yra išlaikomas deformacijų vientisumas. Betonui pasiekus pleišėjimo ribą, jo įtempimai pradeda kristi pagal tam tikrą kreivinę pareinamybę ir palaipsniui mažėja iki nulio. Tikrumoje tempiamos zonos darbą nusako reiškiniai, būdingi ir vienam, ir kitam modeliui. Literatūroje nurodoma, kad antrasis modelis yra ne mažiau realistiškas už pirmąjį, tradicinį modelį.

Darbe pasirenkamas antrasis modelis, kuris yra algoritmiškai patogesnis. Deformacijos čia atvaizduojamos susidedančios iš tiesinės, šiuo atveju, valkšnumo dedamosios, ir netiesinės, pleišėjimo deformacijų dedamosios. Pastaroji čia, kaip ir trumpalaikės apkrovos atveju yra laikoma nepriklausanti nuo laiko. Betonas traktuojamas kaip senėjanti medžiaga, jo valkšnumas yra aprašomas integraline pareinamybe su analitiškai išreikšta valkšnumo funkcija. Skaičiuojamasis laikotarpis yra sudalinamas į eilę laiko žingsnių, integralas pakeičiamas baigtine suma, ir tokiu būdu gaunamos algebrinės pareinamybės tarp įtempimų ir deformacijų prieaugių atitinkamuose laiko intervaluose. Šios pareinamybės priklauso ne tik nuo prieaugių paskutiniame laiko žingsnyje, bet ir nuo visos apkrovimo istorijos. Tai yra neišvengiama senėjančios medžiagos savybė. Kadangi ryšys tarp pleišėjimo deformacijų ir įtempimų yra netiesinis, tai minėtosios prieaugių lygtys sprendžiamos iteraciniu būdu.

Yra sudaryta kompiuterio programa ir sumodeliuotas gelžbetoninio elemento tempiamos zonos valkšnumas, esant įvairiems priverstinės deformacijos didėjimo režimams. Iteracinis procesas parodė gerą konvergenciją. Pasirodė, kad valkšnumas turi palyginus nedidelę įtaką šio modelio pleišėjimo deformacijai. Sudarytosios skaitmeninės procedūros yra skirtos įvesti į baigtinių elementų metodo programą sijų ir rėmų valkšnumui skaičiuoti.

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