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GAME EQUILIBRIUM BASED CONTROL ANALYSIS ON THE SUSTAINABLE MARKET STRUCTURE OF RARE METAL MINERAL RESOURCES – EVIDENCE FROM CHINA

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Abstract. In rare metal mineral market, as a complex system, multiple decision-making among the stakeholders increases the complexity in its market structure and dynamic process. The unreasonable compensation pricing mechanism for the development of the rare metal mineral resources in China requires to be studied. Drawing on the methods of game theory model and chaos control analysis, this paper builds theoretical model of rare metal mineral market structure, corporating related parameters of rare metal in the game theory model, to conduct the chaotic nature and path analysis, expecting to solve the bottleneck problems that restrict the rare metal pricing and resource security and enhance the waste valorization for the sustainability. Specificly, a Cournot-Nash Equilibrium model is built to analyze the Cournot-equilibrium point, the stability of the Cournot Equilibrium point, the chaotic status, as well as the pattern to chaos of the game system in the rare metal mineral resource market, numerical simulation is used to verify the model. The conclusions facilitate the formulation of industrial economic policies and further improvement of managerial strategies to solve market problems.

Keywords: rare metal minerals, multiple decision-making, equilibrium price, complexity analysis, numerical simulation.

Introduction

The rare metal mineral resources are an important strategic material reserve for national security and China has implemented a protective and inhibitory development policy on it for many years, expecting to transform the resource advantages into economic advantages through protective mining (Huang et al., 2014; Zhang et al., 2018). However, in reality, the mineral resources has continued to be extensively exploited and the low-end products that are energy-intensive, highly polluting and resource-dependent have been exported in large quantities. Mineral resource industry policy and resource tax policy have been severely distorted. Furthermore, the disordered chaotic development and vicious competition caused by resource developers' pursuit of short-term interest, low entry barriers and weak technological level have undermined the sellers' monopoly market structure of rare metal mineral resources. As a result, in the actual long-term contract pricing process, the bargaining power of the resources is weak and the pricing power is missing (Zhong et al., 2013; Gong & Lin, 2017, 2018; Liu & Gong, 2020; Li et al., 2019). In the rare-earth industry, supply of low-cost "black industrial chains" with disorderly supply, poor environmental protection and inadequate regulation accounts for nearly 50% of the total supply. Especially after a sharp drop in metal prices in 2015, some copper, tin and nickel enterprises in China have issued statements on production reduction (Chen et al., 2020; Song et al., 2019a; Zhong et al., 2019). However, the existing policy advices for bigger pricing power have neglected the objectivity of strategic value in the process of formulation. Especially for rare metals such as rare earths, lithium and indium, which have no function of price discovery in futures market, the price of international trade is mainly decided by bilateral bargaining, and the bargaining process is influenced by the psychological preference of the game-agents to generate strategic value (Zhong et al., 2013; Jia et al., 2017).

In response to a series of problems existing in the realities of the rare metal mineral resources, scholars have

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carried out a slew of related researches, such as the depletion study of metal mineral resources (Hartwick, 1977; Prior et al., 2012; Northey et al., 2017), the formation mechanism of mineral resource prices (Rubinstein, 1982; Adelman & Watkins, 2008) and the pricing power of rare metal minerals (Alexander et al., 2012). However, the key to solving these problem lies in the accurate analysis of the market structure and the behaviors of various stakeholders. At the academic level, many scholars use the game model to study the interaction of oligopolistic firms in the market (Afflerbach et al., 2014; Tošović et al., 2016). While domestic scholars have combined the game theory with the reality of economy and scored abundant research achievements (Sarjiya et al., 2019; Wang et al., 2019) In recent years, more and more scholars have applied dynamic oligarchic games to specific industrial researches, such as the studies in the electricity market (Ma & Ji, 2009; Zhang et al., 2009). In the field of metal mineral resources, it is also imperative to think about how to solve the chaos faced by the market with new ideas. According to the CR2, CR4 market concentration of such rare metals as rare earths, lithium and indium, their market structure has oligopolistic nature, and each oligopoly makes the same decision simultaneously according to its profit maximization. For example, Tse believes that the rare-earth market belongs to a typical oligopolistic market, and there are oligopolistic game behaviors in the process of rare earths pricing (Tse, 2011). And the simultaneous decision made by various stakeholders adds to the complexity of the market structure and dynamic processes (Chen et al., 2020; Song et al., 2019b; Yang et al., 2017; Lima et al., 2018). Market system is a complex system and for the study of its complexity, many scholars have conducted extensive research on various control problems of different types of control systems with similar structures (Clearwater, 1996). And simulation is a helpful and irreplaceable method for the study of complex system (Begossi, 2014; Tsionas & Michaelides, 2017; Garnier & Solna, 2019).

The above analysis shows that the rare metal minerals market is associated with huge and long-term social and economic benefits, and the impact on specific groups and specific regions is complex and far-reaching. However, determining efficient and fair and reasonable prices is a double theoretical and practical problem. This is because fair and reasonable prices must simultaneously achieve the multiple objectives of "justly protect the loss of mineral resources, ensure that the investment losses of various economic entities are included in the evaluation of the value of mineral resources", and "minimize the cause of inefficient behavior of asset loss". Therefore, in order to reflect the entire cost of the entire society for the rare metal minerals market game and achieve the balance of interests of related parties, it is necessary to redefine the complex value connotation of the metal minerals market game with a new concept and update the quantitative price formation mechanism. The rare metal market system is a complex system, and the simultaneous decision-making of various stakeholders increases the complexity of this market structure and dynamic process. For the purpose of exploring the path and process of China's rare metal market output and price entering chaos, and the game model method and chaos control complex analysis method are drawed to construct a theoretical model of the rare metal mineral market structure, and the actual value of China's rare metal related parameters is incorporated into the complex game model to expand an empirical analysis of the phenomenon and path of chaos in the metal minerals market, so as to solve the bottleneck problem that restricts the pricing power and resource security of rare metal minerals in China.

1. Market structure complexity analysis framework of rare metal mineral resources

1.1. Market structure and stakeholders of rare metal mineral resources

Generally, the market of rare metal mineral resources has more obvious oligopoly characteristics. The key issue in the market structure analysis is to study the market share that affects the market and the price, which depends on the conditions of competition and the probability of substitution. In terms of short-term behavior, there is a lack of price elasticity in the supply of the rare metal minerals. This means that price changes can only cause small linearly proportional changes in production, and that a relatively small market share is enough to dominate the market. Hence, larger producers, whose output accounts for 30–40% of the total supply, can lead to market monopoly.

The allocation of research resources should be based on the market structure. Due to the differences and unequal distributions of mineral resources, the exploration and exploitation of mineral resources are characterized as large in terms of investment scale with long project duration and relatively high investment risk. Also, as the owner of mineral resources, China can intervene in the exploration and exploitation of important mineral resources that are related to national economy and people's livelihood through political means, and promote the transformation from a mineral resource market into one that is monopolistically competitive. In recent years, due to the non-renewability of mineral resources, many multinational enterprise have stepped up their control over the global strategic mineral resources. Take the iron ore market as an example. According to statistics, the international top three companies, Companhia Vale do Rio Doce, BHP Billiton Ltd. and Rio Tinto have controlled over 80% of the world's iron ore resources.

Classical economic theory has determined that monopolistic behavior in the market has a clear restraining effect on the efficient allocation of resources. In the monopolistic market, monopoly producers can make or control the output and price of mineral resources by themselves, and greatly increase the profit of miners, especially excess profits. The monopolist sets a "target rate of profit" that is above the average rate and uses it as a basis for setting the price of mineral resources while adjusting the rate of mining based on its marketing strategy and profit targets to create a more favorable optimal mining condition. There are three forms of realistic monopoly control: maintain the existing state of monopoly; raise the price of mineral resources in order to obtain higher monopoly profits, which, from the perspective of the trend of scarcity, higher price force the manufacturers with lower production efficiency to withdraw from the market. Hence, in terms of protecting scarce resources only, it is of more help to restrict resource depletion; seize market share at low prices. This predatory pricing that aims to crush other competitors and occupy the market requires abundant mineral products, whilch can only be guaranteed by accelerating its mining. Also, consumers may lack awareness of efficient resource utilization because of low prices, causing over-consumption and accelerating the depletion of mineral resources. Hence, under the circumstance of monopoly market, the depletion of ore is affected by many factors and is hard to predict. The change depends on the game results between monopoly developers, between monopolists and consumers and between consumers themselves. And the general market price signals and conditions of supply and demand can not truly reflect the depletion of resources. Then under the assumption of perfectly competitive market, researches on finding the best allocation of mineral resources or the depletion rate(mining rate) through spontaneous market allocation or price lever with value as the center, the supply-demand situation as the influencing factors, has lost its practical natures in the real market of monopolistic competition.

Cournot-Nash equilibrium means that no other manufacturer can choose a different strategy to achieve higher profits while the other manufacturer's strategy remains the same. In the oligopoly market of rare metal mineral resources, each oligarch tends to choose the strategy that is most beneficial to him. The equilibrium reached by this non-collusion behavior between oligarchs is the Cournot-Nash equilibrium. Therefore, it is a better choice to analyze the market of rare metal mineral resources by establishing the Cournot-Nash Equilibrium model.

1.2. The routes to chaos of the game system in the rare metal mineral resources market

The market game system is transformed from deterministic to chaotic movements, which is to say, the routes to chaos in the market game system are divided into severals: the period doubling bifurcation, the intermittence, the Ruelle-Takens route and the bifurcation caused by quasi market game period bifurcation, quasi-periodic attractor fragmentation, multiple attractor coexistence and non-strict periodic increase. The market game chaotic system changes are not in chaos, but rather, in an orderly way. Under the condition of certain market structure, changing the game-related parameters can split the track of the market game system. The market game period is doubled, which means the period is divided into two. The market game parameters continue to change, so dose the split of the market game track. Growing from two to four to eight times of the period, the market game continues, until the game system eventually losses its periodicity and enters the chaos. The basic way of market game movements: Fixed point → 2nd period point → 4th period point → ... → Infinite periodic condensation (limit point) Strange attractor. The process can use logistic mapping as reference to illustrate the route of the period doubling bifurcation to chaos in the market game system:

$$X_n(t+1) = rX_n(t)\left[1 - X_n(t)\right]. \tag{1}$$

This is called a species model, in which $X_n(t) \in [0,1]$, representing the percentage of a species in the market structure and this is in line with the objective reality. The 1st period point after an iteration is the market game periodic solution, which is $X_n(t+1) = X_n(t)$. Because $X_n(t) = rX_n(t)\left[1 - X_n(t)\right]$, the solution to the equation is $X_0 = 0$, $X_1 = 1 - 1/r$. Obviously, the value of r affects the evolutionary process of market game system. Aiming at the different range of the value of *r*, changes from simple to complex state of market game logic mapping is discussed.

(1) When $0 < r < 1$, $X_1 = 1 - 1/r < 0$, it does not meet the condition of $X_n(t) \in [0,1]$, so it has no practical meanings. As for the fixed point, because $\lambda = \left|\frac{\partial X_n(t+1)}{\partial X_n(t)}\right|_{X_0=0} = r < 1$, $X_0 = 0$ is the stable equilibrium point of the logic mapping in the 1- market game period within $[0,1]$.

(2) When $1 < r < 3$, the logic mapping of the market game has two period points, $X_0 = 0$ and $X_1 = 1 - 1/r$, but when $X_0 = 0$, because $\lambda = \left|\frac{\partial X_n(t+1)}{\partial X_n(t)}\right|_{X_0=0} = r > 1$, $X_0 = 0$ is the unstable equilibrium point of the logic mapping in the 1- periodic market game within $[0,1]$. Then, when $X_1 = 1 - 1/r$, because $\lambda = \left|\frac{\partial X_n(t+1)}{\partial X_n(t)}\right|_{X_1=1-1/r} = 2 - r$, and under the condition of $|\lambda| = |2 - r| < 1$, the market game system has only one stable 1- periodic equilibrium point. The evolution of market game system becomes stable at a balance point.

(3) When $3 < r < 1 + \sqrt{6}$, the logic mapping of the market game has two period points, $X_0 = 0$ and $X = 1 - 1/r$, but for $X_0 = 0$, because $\lambda = \left|\frac{\partial X_n(t+1)}{\partial X_n(t)}\right|_{X_0=0} = r > 1$, $X_0 = 0$ is the unstable equilibrium point in the 1- market game period. When

$X_1 = 1-1/r$,because $\lambda = \left|\frac{\partial X_n(t+1)}{\partial X_n(t)}\right|_{X_1=1-1/r} = 2-r < -1a$, we can get $|\lambda| = |2-r| > 1$. Therefore $X_1 = 1-1/r$ is also the unstable equilibrium point in the 1- market game period. At this point the market game system does not have a stable periodic solution. Considering the fixed point after 2 iterations, that is, the 2- periodic solution, after two iterations, the market game system is mapped as:

$$X_n(t+2) = rX_n(t+1)\left[1-X_n(t+1)\right] = r^2 X_n(t)\left[1-rX_n(t)(1-X_n(t))\right]. \quad (2)$$

Suppose that $X_n(t+2) = X_n(t)$, then there are 4 equilibrium points in the market game system at this moment: $X_0 = 0$, $X_1 = 1-1/r$, $X_4 = \frac{1+r-\sqrt{(1+r)(3+r)}}{2}$. And among the four equilibrium points, both $X_0 = 0$ and $X_1 = 1-1/r$ are the unstable equilibrium points of the market game system, while $X_3 = \frac{1+r+\sqrt{(1+r)(3+r)}}{2}$ and $X_4 = \frac{1+r-\sqrt{(1+r)(3+r)}}{2}$ are the stable ones. Therefore, the market game system has two 2- periodic solutions.

1.3. Market game system chaos control

Market game system chaos control refers to the human interference in the chaos system of the market game system, transforming it into a required state. Specifically speaking, when the chaotic movement in the market game system is harmful, the chaos in the market game system is suppressed and avoided; when the chaotic movement in the market game system is conducive, various conditions that can generate the chaotic movement in the market game system are created, so as to produce chaos; when the market game system is in the state of chaos, all kinds of outputs needed are generated by the chaos control. For the chaos control, there are the following methods: continuous feedback control method, adaptive control method, neural network method, periodical exciting force method, parametric periodic perturbation method, OPF control method, state feedback and parameter adjustment control method and the method of pulse feedback of systematic variable. In this study, the chaos control method of state feedback and parameter adjustment is specifically disscussed.

The state feedback and parameter adjustment control method of the market game system can effectively implement the delay control on the doubling period bifurcation of the discrete nonlinear dynamic system in the market game system. Taking the market game system n-dimensional discrete nonlinear dynamic system as an example, supposing the n-dimensional discrete nonlinear system equation is as follows:

$$X_{t+1} = g(x_t, \alpha)_Z . \quad (3)$$

In the above equation, $x_t \in R^n$, $t \in Z$, $\alpha \in R$ are the bifurcation parameters of the market game system, that is, the state of the market game system will change as the game variable of the market changes. With the increase of the market game variable, the evolution trajectory goes through the doubling period bifurcation till the market game system reaches the state of chaos. The method of parameter adjustment and the state feedback control is applied to the market game variables to control the output and price strategy, and the controlled market game system is:

$$x_{t+1} = \beta g^m(x_t, \alpha) + (1-\beta)x_t , \quad (4)$$

β is the market game (output and price) variable adjustment parameter, ranging: $0 < \beta < 1$: However, when the adjustment parameter of the market game (output and price) is $\beta = 1$, the controlled $x_{t+1} = \beta g^m(x_t, \alpha) + (1-\beta)x_t$ degenerates to $X_{t+1} = g(x_t, \alpha)$. At this point, the market game (output and price) system has the same positive integer, the m periodic orbit, among which, $g^m(\bullet)$ is the m th power complex function of function $g(\bullet)$. The parameter m represents the m-period orbit controlled by the market game system: when $m = 1$, the market game system controls the equilibrium point, and the discrete power system of the market game system adds one control every iteration; when $m > 1$, the market game system adds one control every m iterations. When $m = 1$, assume that in the original system $X_{t+1} = g(x_t, \alpha)$, its market equilibrium point is x^*, then the Jacobian matrix at market equilibrium point x^* is:

$$J_1 = \left|\frac{\partial g(x_t, \alpha)}{\partial x_t}\right|_{x_t = x^*} . \quad (5)$$

The sufficient and necessary condition for the market equilibrium point x^* to be stable is that the modulus of all eigenvalues of the Jacobian matrix J_1 is less than 1, that is $|i| < 1 (i = 1, 2, \cdots, n)$. From this, the range of the market variable adjustment parameter α of the original system $X_{t+1} = g(x_t, \alpha)$ can be obtained when the market equilibrium point x^* is stable. The Jacobian matrix of controlled system $x_{t+1} = \beta g^m(x_t, \alpha) + (1-\beta)x_t$ at market equilibrium point x^* is:

$$J_2 = \left|\beta \frac{\partial g(x_t, \alpha)}{\partial x_t} + (1-\beta)\right|_{x_t = x^*} . \quad (6)$$

As Jacobi's matrix J_2 introduces a variable parameter β that adjusts the market strategy, by appropriately assigning the market strategy variable parameter β, market stability at the equilibrium point x^* within a larger value range of the variable parameters in the strategy choice of the bifurcation market can be achieved. Hence, the market game system bifurcation and chaos will be delayed. The $x_{t+1} = \beta g^m(x_t, \alpha) + (1-\beta)x_t$ system can still ensure that the modulus of all eigenvalues are less than 1 (the eigenvalues of $J_2 : |i| < 1 (i = 1, 2, \cdots, n)$) even at the unstable market equilibrium point x^* of the original system $X_{t+1} = g(x_t, \alpha)$ within the value range of α. That is to say, the market equilibrium point x^* of system $x_{t+1} = \beta g^m(x_t, \alpha) + (1-\beta)x_t$ can remain stable when it is unstable in the system $X_{t+1} = g(x_t, \alpha)$ within the value range of α, thus delaying the market game system bifurcation and chaos.

2. Analyses of the structual complexity of rare metal mineral resource market

Based on structual analysis of rare metal mineral resource market, the Chaotic Theory and the Theory of Game will be combined in the following part so as to carry out complexity analysis to the repetitive price game of oligarch market featuring Cournot-Nash equilibrium. Also, by changing the cost and demand function of rare metal mineral resource market and adding factors such as bonded rationality, changes will be made to the Cournot-Nash game model for the aim of finding out the relation between output and market phenomena such as bifurcation and chaos in Cournot-Nash repetitive game.

2.1. The construction of dual oligarch Cournot-Nash model of game theory of rare metal mineral resources

Given the dual oligarch Cournot model of game theory of rare metal mineral, q_1 stands for the supply of the products of the rare metal mineral resource developers, $i = 1,2$. In the oligopoly market of a rare metal mineral resources, developers provide rare metal mineral resources in the discrete time period of t $(t = 0,1,2,\cdots)$ and the output of the mineral resources of i in the period t is marked as $q_1(t)$. In order to get the maximum output of the mineral resources in $t+1$, deverlopers of rare metal minerals must come up with a prediction of the output of their opponents in one development cycle, hence the total output of the metal mineral resource market is as follows:

$$Q(t) = q_1(t) + q_2(t). \tag{7}$$

And the inverse demand function of metal mineral resources market is:

$$P = p(Q(t)) = m - nQ(t). \tag{8}$$

The assumption of the costs of two rare metal mineral resource developers is:

$$c_i = r_i + \eta_i q_i, \tag{9}$$

$c_i(i = 1,2)$ stands for the costs of two rare metal mineral resource developers and r_i is their fixed cost, then $\eta_i q_i$ is the variable costs of two developers respectively $(t_i > 0)$.

Thus, the profits of rare metal mineral resource developer 1 in period t are:

$$\pi_1(t) = q_1(t)\left[m - n\left(q_1(t) + q_2(t)\right)\right] - \left(r_1 + \eta_1 q_1\right). \tag{10}$$

And the profits of rare metal mineral resource developer 2 in period t are:

$$\pi_2(t) = q_2(t)\left[m - n\left(q_1(t) + q_2(t)\right)\right] - \left(r_2 + \eta_2 q_2\right). \tag{11}$$

By taking the first derivative with respect to the revenue functions of rare metal mineral resource developers, the marginal profits of rare metal mineral resource developers are as follows:

$$\frac{\partial \pi_1(t)}{\partial q_1(t)} = m - 2nq_1(t) - nq_2(t) - \eta_1; \tag{12}$$

$$\frac{\partial \pi_1(t)}{\partial q_2(t)} = m - 2nq_2(t) - nq_1(t) - \eta_2. \tag{13}$$

In the actual market of rare metal mineral resources, the games between developers are perpetual, that is why the decision-making of developers'output policies is a dynamic process. Developers will put forward output and price policies based on the result of previous games. If they were satisfied with the results of last game, they might continue to use the previous stragegy, believing this strategy will secure a good result. For all the rare metal mineral resource developers, the ultimate goal is the same, that is to say, to get the maximum of profits. Therefore, marginal profit is a key indicator of choosing game strategies of output and price. If the marginal profit of last round of game is greater than zero, developers tend to continue to implement the previous price strategy because the profit will keep on rising. Thus we have the repeated price model of game theory:

$$q_i(t+1) = q_i(t) + \alpha_i q_i(t)\frac{\partial \pi_i}{\partial q_i}, \tag{14}$$

$\alpha_i(i = 1,2)$ is the intensity and speed of output adjustment, and by putting $\frac{\partial \pi_i}{\partial q_i}(i = 1,2)$ into the equation above, the dual oligarch repeated output models of game theory are as follows:

$$q_1(t+1) = q_1(t) + a_1 q_1(t)\left(m - 2nq_1(t) - nq_2(t) - \eta_1\right), \tag{15}$$

$$q_2(t+1) = q_2(t) + a_2 q_2(t)\left(m - 2nq_2(t) - nq_1(t) - \eta_2\right). \tag{16}$$

2.2. The equilibrium point and stability analysis in the oligarch market of rare metal minerals resources

The game system of rare metal mineral resouces shows that developers will make adjustments to their output based on the marginal profits in the last round of game. If the marginal profit in period t is greater than zero, developers will continue to use the output strategy to set the price in period $t+1$, even the adjustment intensity and speed of rare metal mineral output are different in period $t+1$, which will affect the outcome of the output game between rare metal mineral developers. Therefore, the adjustment intensity and speed of the output strategy of rare metal mineral developers, that is, α_i in the model is the main factor affecting the price game of rare metal mineral developers.

According to $q_i(t+1)=q_i(t)$, four system balance points can be obtained:

$$E_1=(0,0), E_2=\left(0,\frac{m-\eta_2}{2n}\right), E_3=\left(\frac{m-\eta_1}{2n},0\right);$$

$$E_4=\left(\frac{m-2\eta_1+\eta_2}{3n},\frac{m-2\eta_2+\eta_1}{3n}\right).$$

Jacobian matrix:

$$J=\begin{pmatrix} J_{11} & J_{12} \\ J_{21} & J_{22} \end{pmatrix}=\begin{pmatrix} 1+\alpha_1\left(m-4nq_1(t)-nq_2(t)-\eta_1\right) & -n\alpha_1 q_1(t) \\ -n\alpha_2 q_2(t) & 1+\alpha_2\left(m-4nq_2(t)-nq_1(t)-\eta_2\right) \end{pmatrix}.$$

1) For $E_1=(0,0)$, the corresponding Jacobian matrix is:

$$J(E_1)=\begin{pmatrix} 1+\alpha_1(m-\eta_1) & 0 \\ 0 & 1+\alpha_2(m-\eta_2) \end{pmatrix}.$$

Because of $m>\eta_1$, $m>\eta_2$, $\alpha_i>0\ (i=1,2)$, the two characteristic roots of $J(E_1)$ are $\lambda_1=1+\alpha_1(m-\eta_1)>1$, $\lambda_2=1+\alpha_2(m-\eta_2)>1$. Therefore, the equilibrium point $E_1=(0,0)$ is an unstable point.

2) For $E_2=\left(0,\frac{m-\eta_2}{2n}\right)$, the corresponding Jacobian matrix is:

$$J(E_2)=\begin{pmatrix} 1+\frac{\alpha_1(m-2\eta_1+\eta_2)}{2} & 0 \\ -\frac{\alpha_2(m-\eta_2)}{2} & 1-\alpha_2(m-\eta_2) \end{pmatrix}.$$

A characteristic roots of $J(E_2)$ is $\lambda_1 = 1 + \frac{\alpha_1(m-2\eta_1+\eta_2)}{2}>1$, therefore the equilibrium point $E_2=\left(0,\frac{m-\eta_2}{2n}\right)$ is an unstable point.

3) For $E_3=\left(\frac{m-\eta_1}{2n},0\right)$, the corresponding Jacobian matrix is:

$$J(E_3)=\begin{pmatrix} 1-\alpha_1(m-\eta_1) & -\frac{\alpha_1(m-\eta_1)}{2} \\ 0 & 1+\frac{\alpha_2(m+\eta_1-2\eta_2)}{2} \end{pmatrix}.$$

A characteristic roots of $J(E_3)$ is $\lambda_2=1+\frac{\alpha_2(m+\eta_1-2\eta_2)}{2}>1$, therefore the equilibrium point $E_3=\left(\frac{m-\eta_1}{2n},0\right)$ is an unstable point.

4) For $E_3=\left(\frac{m-\eta_1}{2n},0\right)$, the corresponding Jacobian matrix is:

$$J(E_4)=\begin{pmatrix} \frac{1}{3}\left(3-2\alpha_1(m-2\eta_1+\eta_2)\right) & -\frac{1}{3}\alpha_1(m-2\eta_1+\eta_2) \\ -\frac{1}{3}\alpha_2(m+\eta_1-2\eta_2) & \frac{1}{3}\left(3-2\alpha_2(m+\eta_1-2\eta_2)\right) \end{pmatrix}.$$

The characteristic polynomial of $J(E_4)$: $F(\lambda)=\lambda^2-Tr\lambda+Det$, $\Delta=Tr^2-4Det>0$. This means that the characteristic polynomial of the expected output E of rare metal minerals has real characteristic roots. Therefore, according to Jury's argument for determining stability based on the Nash equilibrium point of discrete systems (Wang et al., 2019; Zhong et al., 2019), the point is locally stable if:

$$c: Det-1<0.$$

That is:

$$\begin{cases} (m+\eta_1-2\eta_2)(m-2\eta_1+\eta_2)>0 \\ -4\left[-3+\alpha_2(m+\eta_1-2\eta_2)\right]+\alpha_1\left[-4+\alpha_2(m+\eta_1-2\eta_2)\right]\times (m-2\eta_1+\eta_2)>0 \\ -2\alpha_2(m+\eta_1-2\eta_2)+\alpha_1\left[-2+\alpha_2(m+\eta_1-2\eta_2)\right]\times (m-2\eta_1+\eta_2)<0 \end{cases}$$

Let:

$$f(\alpha_1)=-4\left[-3+\alpha_2(m+\eta_1-2\eta_2)\right]+\alpha_1\left[-4+\alpha_2(m+\eta_1-2\eta_2)\right](m-2\eta_1+\eta_2);$$

$$g(\alpha_1)=-2\alpha_2(m+\eta_1-2\eta_2)+\alpha_1\left[-2+\alpha_2(m+\eta_1-2\eta_2)\right](m-2\eta_1+\eta_2).$$

From $f(\alpha_1)=0$, $g(\alpha_1)=0$, two roots can be obtained as:

$$\alpha_1^*=\frac{4(-3+\alpha_2 m+\alpha_2\eta_1-2\alpha_2\eta_2)}{(m-2\eta_1+\eta_2)(\alpha_2 m+\alpha_2\eta_1-2\alpha_2\eta_2-4)};$$

$$\alpha_2^*=\frac{2\alpha_2(m+\eta_1-2\eta_2)}{(m-2\eta_1+\eta_2)(\alpha_2 m+\alpha_2\eta_1-2\alpha_2\eta_2-2)};$$

$$\alpha_1^* - \alpha_2^* = \frac{2[(\alpha_2(m+\eta_1-2\eta_2)-3)^2+3]}{[\alpha_2(m+\eta_1-2\eta_2)-4][\alpha_2(m+\eta_1-2\eta_2)-2](m-2\eta_1+\eta_2)}.$$

Assume that: $d=\alpha_2(m+\eta_1-2\eta_2)-4$, $e=\alpha_2(m+\eta_1-2\eta_2)-2$. Because $m+\eta_1-2\eta_2>0$ and $m-2\eta_1+\eta_2>0$, $1-Tr+Det>0$.

① When $\alpha_2 < \frac{2}{m+\eta_1-2\eta_2}$, $d<e<0$ and $\alpha_1^{**}<0<\alpha_1^*$. Solution condition *b* is $\alpha_1<\alpha_1^*$, solution condition *c* is $\alpha_1>\alpha_1^{**}$. So when $\alpha_1\in(0,\alpha_1^*)$, the jury condition is satisfied, at which point $E_4=\left(\frac{m-2\eta_1+\eta_2}{3n},\frac{m-2\eta_2+\eta_1}{3n}\right)$ is a stable point. The feasible regions of α_1 and α_2 are shown in the black part of Figure 1.

② When $\frac{2}{m+\eta_1-2\eta_2}<\alpha_2<\frac{3}{m+\eta_1-2\eta_2}$, $d<0<e$ and $0<a_1^*<a_2^*$. Solution condition *b* is $\alpha_1<\alpha_1^*$, solution condition *c* is $\alpha_1<\alpha_1^{**}$. So when $\alpha_1\in(0,\alpha_1^*)$, the jury condition is satisfied, at which point $E_4=\left(\frac{m-2\eta_1+\eta_2}{3n},\frac{m-2\eta_2+\eta_1}{3n}\right)$ is a stable point. The feasible regions of α_1 and α_2 are shown in the grey part of Figure 1.

③ When $\frac{3}{m+\eta_1-2\eta_2}<\alpha_2<\frac{4}{m+\eta_1-2\eta_2}$, $d<0<e$ and $a_1^*<0<a_2^*$. Solution condition *b* is $\alpha_1<\alpha_1^*$, which is contradictory, therefore at this point $E_4=\left(\frac{m-2\eta_1+\eta_2}{3n},\frac{m-2\eta_2+\eta_1}{3n}\right)$ is not a stable point. So the feasible regions of α_1 and α_2 does not exist.

④ When $\alpha_2>\frac{4}{m+\eta_1-2\eta_2}$, $0<d<e$ and $a_1^*>a_2^*$. By bringing in conditions *b* and *c* it is easy to prove that there is no solution, at which point $E_4=\left(\frac{m-2\eta_1+\eta_2}{3n},\frac{m-2\eta_2+\eta_1}{3n}\right)$ is not a stable point. So the feasible regions of α_1 and α_2 does not exist.

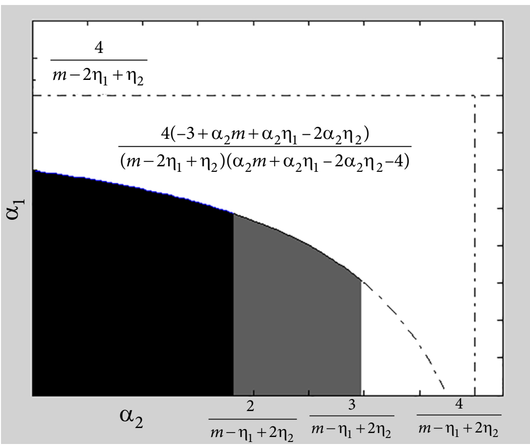

Figure 1. The stable area of Cournot-Nash Equilibrium of rare metal mineral development

According to the stability analysis, the value of the variable α_i will not affect the value of the Cournot equilibrium point of the rare metal mineral developer, but will affect the stability of the Cournot equilibrium point. The theoretical significance of the stable area of the Cournot equilibrium point of the rare metal mineral developer: When the output adjustment variable (α_1,α_2) falls within the stable area, the Cournot equilibrium of the rare metal mineral developer is stable. If the output adjustment variable (α_1,α_2) exceeds the stable domain, the rare metal mineral developer game system will become unstable and will undergo chaotic movement. This theoretical model reflects the practical significance of the market structure of rare metal mineral developers: In the actual rare metal minerals market game competition, before the output of the rare metal mineral developers has reached the Cournot equilibrium, the rare metal mineral developers will continue to conduct output games in order to obtain more economic profits, that is, constantly adjust their respective output. When the developers of rare metal minerals do not adjust the output fast and fall within the stable range, the market of rare metal minerals is relatively stable. If the output adjustment speed of any one of the developers of rare metal minerals is too fast and exceeds the stable area of the Cournot equilibrium, the entire rare metal minerals market will be in a chaotic state, the rare metal minerals market will be volatile, the price of rare metal minerals will fluctuate greatly, and the competition in the metal and mineral industry is fierce. To a certain degree of competition, the state needs macro-control to maintain the stability of the rare metal mineral market. Many countries will issue mining policies, resource tax policies, mineral development environmental policies and mineral export policies when the market is confined to a chaotic state to regulate the domestic rare metal mineral market and let the market operate in a stable state.

2.3. The stability analysis of Cournot-Nash equilibrium in the oligarch market of rare metal mineral resources

Considering the current status of rare metal market and the number of gaming oligarchs in the rare metal resource market (Chen et al., 2016) and the characteristics of Cournot-Nash equilibrium (Zhong et al., 2016; Jia et al., 2017), this study assigns the parameters in the game system of rare metal minerals: $m=5$, $n=1$, $\eta_1=0.1$, $\eta_2=0.2$, the results are as follows:

$$q_1(t+1)=q_1(t)+\alpha_1 q_1(t)(4.9-2q_1(t)-q_2(t)); \quad (17)$$

$$q_2(t+1)=q_2(t)+\alpha_2 q_2(t)(4.8-2q_2(t)-q_1(t)). \quad (18)$$

By setting the anticipated output as $q_i(t+1) = q_i(t)$, we have:

$$\alpha_1 q_1(t)(4.9 - 2q_1(t) - q_2(t)) = 0; \tag{19}$$

$$\alpha_2 q_2(t)(4.8 - 2q_2(t) - q_1(t)) = 0. \tag{20}$$

After solving the equation above by using Matlab, the non-negative real root is $E = (q_1 = 1.667, q_2 = 1.567)$. In order to secure the stability of the equilibrium point in the game of rare metal mineral resources, the author has taken the Jacobian Matrix (10) of the system into consideration:

$$J = \begin{pmatrix} 1+\alpha_1(4.9-4q_1-q_2) & -\alpha_1 q_1 \\ -\alpha_2 q_2 & 1+\alpha_2(4.8-4q_2-q_1) \end{pmatrix}. \tag{21}$$

By putting the value of E into the characteristic Eq. (13), two characteristic modules of the equation are less than 1, so E is the Cournot-Nash equilibrium of the system (10). The marginal profit of two developers is zero at the Cournot-Nash equilibrium, but it does not mean that the game is stabalized. If one developer changes the output, the process of the game can be extremely complicated. So the Cournot-Nash equilibrium only suggests that the game is partially stable and the stable region is determined by (α_1, α_2) of developers. In order to find out the stable region of Cournot-Nash equilibrium, the author put E into (13) and get the characteristic polynomial of it:

$$F(\lambda) = \lambda^2 - Tr\lambda + Det = 0. \tag{22}$$

In this polynomial:

$$\begin{aligned} Tr &= 1+\alpha_1(4.9-4q_1-q_2)+1+\alpha_2(4.8-4q_2-q_1) = \\ &1+\alpha_1(4.9-4\times1.667-1.567)+1+\alpha_2(4.8-4\times1.567-1.667) = \\ &2-3.335\alpha_1-3.135\alpha_2; \end{aligned} \tag{23}$$

$$\begin{aligned} Det &= \left[1+\alpha_1(4.9-4q_1-q_2)\right]\left[1+\alpha_2(4.8-4q_2-q_1)\right] - \\ &\alpha_1\alpha_2 q_1 q_2 = (1-3.335\alpha_1)(1-3.135\alpha_2) - 2.612\alpha_1\alpha_2. \end{aligned} \tag{24}$$

By putting Tr and Det into the characteristic polynomial , the Δ is:

$$\begin{aligned} \Delta &= Tr^2 - 4Det == (2-3.335\alpha_1-3.135\alpha_2)^2 - \\ &4\times\left[(1-3.335\alpha_1)(1-3.135\alpha_2)-2.612\alpha_1\alpha_2\right] = \\ &\left[(1-3.335\alpha_1)-(1-3.135\alpha_2)\right]^2 + 10.455\alpha_1\alpha_2. \end{aligned} \tag{25}$$

Obviously, the Δ is greater than zero so that the characteristic polynomial of E has real characteristic root. Therefore, in this thesis the stable region of Cournot-Nash equilibrium can be determined according to the condition of Jury:

$$\begin{aligned} &a: 1 - Tr + Det > 0, \\ &b: 1 + Tr + Det > 0, \\ &c: Det - 1 < 0. \end{aligned} \tag{26}$$

That is:

$$\begin{aligned} &a: 1-(2-3.335\alpha_1-3.135\alpha_2)+ \\ &(1-3.335\alpha_1-3.135\alpha_2+10.455\alpha_1\alpha_2) > 0; \\ &b: 1+(2-3.335\alpha_1-3.135\alpha_2)+ \\ &(1-3.335\alpha_1-3.135\alpha_2+10.455\alpha_1\alpha_2) > 0; \\ &c: (1-3.335\alpha_1-3.135\alpha_2+10.455\alpha_1\alpha_2) - 1 < 0. \end{aligned} \tag{27}$$

By solving inequations above, we can get the stable region of Cournot-Nash equilibrium, which is determined by variables (α_1, α_2), α_1 and α_2 stand for horizontal and vertical axis respectively. According to Figure 2, the intersections of the bifurcation curve and two axes are (0.6,0) and (0,0.638).

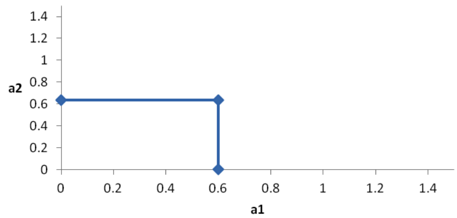

Figure 2. Stable region of Cournot-Nash equilibrium of rare metal mineral resources

According to the analysis of stability, the value of α_i will not affect the value of Cournot-Nash equilibrium, but it will exert influence to the stability of developers' Cournot-Nash equilibrium. The implication of the stable region of Cournot-Nash equilibrium of rare metal mineral resources is that when (α_1, α_2), the variable of output adjustment, is included in the stable region, the Cournot-Nash equilibrium is stable; when is beyond the stable region, the Cournot-Nash equilibrium is unstable and chaotic motion will occur. The practical meaning of this model in rare metal mineral resource market: in the actual game of rare metal mineral resources market, in order to get more profits, developers will continuously adjust their output until it reaches Cournot-Nash equilibrium. When the rate of adjustment is slow and included in the stable region, the market will be comperatively stable. If any one of those developers increases the rate of adjustment so fast that it goes beyond the stable region, turbulance and fierce competition will occure and the price of rare metal mineral resources will fluctuate greatly. The competition will be so fierce that the central government will have to take measures to stabalize the market by means of macro regulation. Next, the author will anaylze how the game system of the market will be affected by the change of α_i.

2.4. Numerical simulation of the stability of Cournot-Nash equilibrium in dual oligarch market of rare metal mineral resources

As the (α_1, α_2) goes beyond the stable region, the stability of Cournot-Nash equilibrium will start to change. In Figure 3 and Figure 4, it is the process of Cournot-Nash game system entering chaotic state through double period bifurcation. In Figure 3, when $\alpha_1 = 0.56$, as α_2, the rate of output adjustment increases, the curve of developers'output changes. In this research, when $\alpha_2 < 0.15$, Cournot-Nash equilibrium is stable (in the first period). However, when $\alpha_2 > 0.15$, the stability changes, and the output starts double period bifurcation, hence entering the second period. As α_2 increases, the output of rare metal mineral resources enters chaotic state through double period bifurcation. Figure 4 shows the systemic bifurcation caused by the increase of α_1, when $\alpha_2 = 0.12$ when $\alpha_2 < 0.58$, Cournot-Nash equilibrium is stable (in the first period). However, when $\alpha_2 > 0.58$, the stability changes, and the output starts double period bifurcation, hence entering the second period. As α_2 increases, the output of rare metal mineral resources enters chaotic state through double period bifurcation. Figure 5 and Figure 6 illustrate the chaotic attractors provided that $(\alpha_1 = 0.56, \alpha_2 = 0.55)$ and $(\alpha_1 = 0.56, \alpha_2 = 0.6)$.

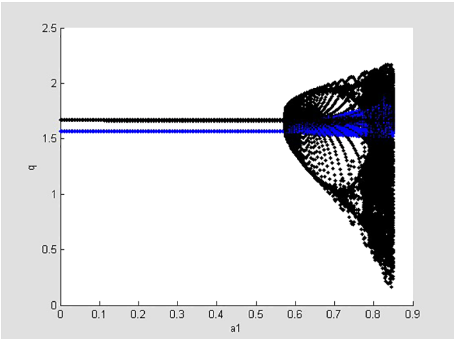

Figure 3. $\alpha_1 = 0.56, \alpha_2 \in [0, 0.6]$

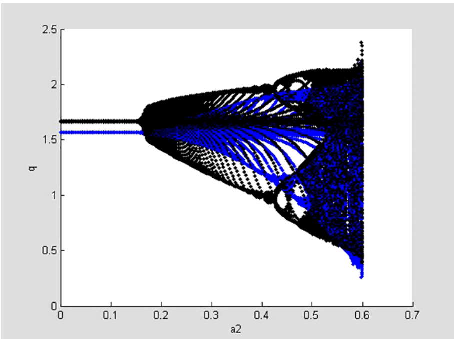

Figure 4. $\alpha_1 = 0.12, \alpha_2 \in [0, 0.85]$

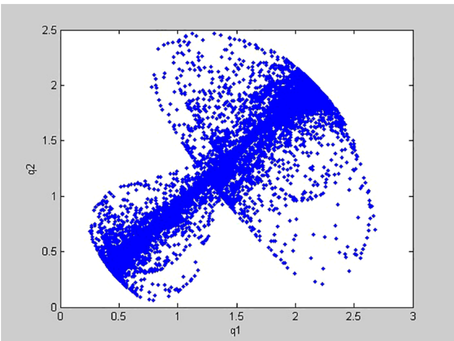

Figure 5. $(\alpha_1 = 0.56, \alpha_2 = 0.55)$

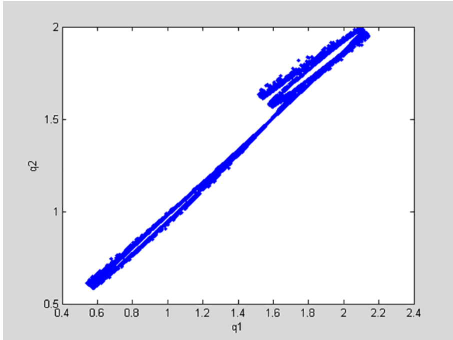

Figure 6. $(\alpha_1 = 0.56, \alpha_2 = 0.6)$

2.5. Chaos control of the stability of Cournot-Nash equilibrium in the dual oligarch market of rare metal mineral resources

Judging from the analysis of stability and chaos of Cournot-Nash equilibrium in the dual oligarch market of rare metal mineral resources, chaos will bring turbulance and instability to the market, so chaos is less favored in the market, hence calling for control of chaos. There are two aspects of chaos control in the market: if the chaos is harmful to the market, proper measures should be adopted to prevent chaos in the game system from happening; if the chaos is conducive to the market, lead and control will be taken in order to create chaos in the game system of rare metal mineral resource market. That is to say, for all the stakeholders in the market, to take control on chaos means that they will adopt measures to avoid chaos so as to maintain the equilibrium of output in the market. To control chaos in the game system of the rare metal mineral resource market, control strategies such as giving feedbacks of variables and adjusting parameters in the game system can be adopted based on the characteristic of the rare metal mineral resource market. To be specific, the equation of game system is:

$$q_1(t+1) = X[q_1(t), q_2(t)];$$
$$q_2(t+1) = Y[q_1(t), q_2(t)]. \quad (28)$$

Controlled system of rare metal mineral resource market:

$$q_1(t+1) = (1-\gamma)X^n[q_1(t), q_2(t)] + \gamma q_1(t);$$
$$q_2(t+1) = (1-\gamma)Y^n[q_1(t), q_2(t)] + \gamma q_2(t). \quad (29)$$

Here, γ is the parameter of control in the game system $(\gamma \in [0,1])$, it can postpone the occurance of bifurcation in the market of rare metal mineral resources. When $\gamma = 0$, the controlled system is the original system. In the original system, n stands for the track of period n. When $n = 1$, known as the Cournot-Nash equilibrium, the controlled game system is:

$$q_1(t+1) = (1-\gamma)(q_1(t) + \alpha_1 q_1(t)(4.9 -$$
$$2q_1(t) - q_2(t))) + \gamma q_1(t);$$
$$q_2(t+1) = (1-\gamma)(q_2(t) +$$
$$\alpha_2 q_2(t)(4.8 - 2q_2(t) - q_1(t))) + \gamma q_2(t). \quad (30)$$

By giving certain value to γ, the stability of Cournot-Nash equilibrium can be maintained in a larger value range of the bifurcation parameter. For the sake of research, this study assigns $\gamma = 0.25, \alpha_1 = 0.56$, and the values of other parameters stay consistent with ones in the numerical simulation, the controlled system (3–5) of market of rare metal mineral resources is as follows:

$$q_1(t+1) = 0.75\Big(q_1(t) + \alpha_1 q_1(t)\big(4.9 - 2q_1(t) - q_2(t)\big)\Big) + 0.25q_1(t);$$

$$q_2(t+1) = 0.75\Big(q_2(t) + \alpha_2 q_2(t)\big(4.8 - 2q_2(t) - q_1(t)\big)\Big) + 0.25q_2(t). \tag{31}$$

Compared with Figure 3, the stable region of Cournot-Nash equilibrium in the controlled system in Figure 7 has been enlarged, and α_2, the bifurcation point in controlled system, has increased from 0.15 in the original system to 0.55. That is to say, after the game system of market of rare metal mineral resources is controlled, the double perioed bifurcation is postponed so that the chaos in the game is prevented.

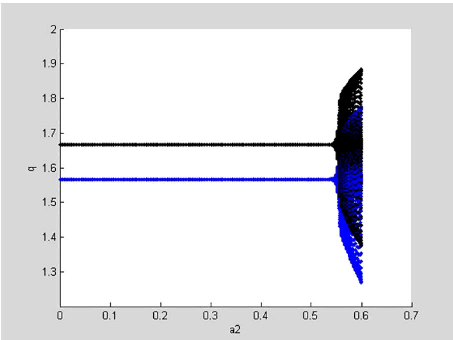

Figure 7. Bifurcation in the game system of the rare metal mineral resources market $\alpha_1 = 0.56, \alpha_2 \in [0, 0.6]$

Based on analyses above, the decisions of developers definitely serve for the pursuit of maximum profit, so they will continuously adjust output to reach the maximum profit. As the rate of adjustment increases, the stability of the game system changes as well. In this thesis, the author found that once the rate of adjustment (α_1, α_2) goes beyond the stable region of Cournot-Nash equilibrium, chaos will occur in the game system. This finding can explain the reason why intensified competition between developers can end up being chaotic in actual market theoretically. Rare metal minerals are key raw materials for the implementation of major national needs such as "Made in China 2025" and "Revolutionary Strategy of Energy Production and Consumption (2016–2030)", thus its fluctuation will inevitably impact the whole economy. The chaos in the market is what the state do not want. Therefore, the state must influence the behavior of developers through resource tax, environmental tax and industrial policy. These developers should make proper regulation to the rate of price adjustment respectively so as to keep (α_1, α_2), the rate of change in the stable region.

Conclusions

Based on the game model of market structure and the theoretical analysis framework of complexity, this paper constructs the Cournot-Nash Game theory model of the rare metal mineral market. Combined with the actual situation of China's rare metal market structure, the Cournot equilibrium point, Cournot equilibrium point stability and Cournot equilibrium stable chaos state and the path of chaos in China's rare metal market are analyzed. Specific conclusions are as follows:

(1) Using the data in China, the simulated rare metal mineral production strategy variable (α_1, α_2) falls between the interval (0.6, 0) and (0, 0.638).

(2) Only when (α_1, α_2), variable of output adjustment, remains in the stable region, the Cournot-Nash equilibrium of developers is stable; when (α_1, α_2) goes beyond the stable region, the game system of developers will become unstable and chaotic motion will occur.

(3) When the output adjustment variable (α_1, α_2) of the rare metal mineral developer breaks out of the Cournot equilibrium stable area, the stability of the Cournot Nash equilibrium will change, and the Cournot game system will eventually enter the chaotic process after a period-doubling bifurcation.

(4) The chaotic phenomenon of the rare metal minerals market game can be suppressed. The doubling cycle bifurcation can be delayed by controlling the rare metal mineral market game system.

According to the above conclusions, the control strategy of state feedback and parameter adjustment of the game system variables in the rare minerals market can control the chaos system of the game in the rare minerals market, and finally make the price of rare metals mineral developers stable at the cournot equilibrium price. The results were consistent with Du et al. (2017), He and Chen (2017), Zhong et al. (2016), and Jia et al. (2017). This conclusion will undoubtedly contribute to the healthy, stable and sustainable development of rare mineral market. The strategies for controlling the rare metals market game in the new era background to avoid entering chaos are as follows:First, the central government should gather some paces in terms of supply side structual reform of rare metal mineral resources, deal with overcapacity in the industry, and carry out M&A, transformation, relocation of outdated capacity in the industry using market methods. Second, great efforts should be made to accomplish joint reorganizaion of rare metal mineral resource enterprises, and the advantages of finance, management, technics of big enterprises in the industry should be brought to full use for the purpose of establishing big conglomorates with huge international competitiveness. Third, the task of joint merger and acquisition should goes hand in hand with methods of eliminating outdated capacity so as to promote the level of product structure and increase the competitiveness of rare metal mineral resource enterprises. Last but not least, Chinese rare metal mineral resource industry should focus on technological innovations, so as to strengthen the competitiveness of products, meet the market demand, and optimize capacity structure from supply side.

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Conflict of interest

The authors declare no competing financial interest.

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