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NUMERICAL METHOD FOR 2D SOLITON SOLUTION AT SHG IN MEDIA WITH TIME-DEPENDENT COMBINED NONLINEARITY

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Abstract. The paper describes the iteration method for finding the eigenfunctions and eigenvalues of the system of two nonlinear Schrödinger equations, which describes the process of second harmonic generation by femtosecond pulse in media with the quadratic and cubic nonlinear response. Coefficients, which characterize the nonlinearities, depend on one of the coordinate. The discussed method allows to find soliton solutions of new form corresponding to the first and second eigenvalues for the wide range of the nonlinear coefficients values. For determination of the eigenfunctions of the third and higher order it is necessary to select the initial approximation in a special way.

Key words: nonlinear Schrödinger equations, eigenfunctions and eigenvalues, laser femtosecond pulse, soliton.

1 Introduction

Investigation of the propagation of femtosecond pulses in different media is of great practical interest due to their short duration and high intensity. In particular, soliton regimes of optical pulses propagation present the great significance for the information technologies by using the optical fiber. Because of the unique properties of solitons they are constantly attract the attention in the literature. As it is well known, soliton is a name for a solitary wave, which does not change in the direction of propagation of optical pulse in medium. Modern laser equipments make it possible to realize the so-called colored solitons, when optical waves at several frequencies exist and propagate simultaneously along the nonlinear medium [2, 3, 4, 5, 6, 9, 10, 11, 12, 13, 14, 15, 16, 17].

The interest to these solitons in literature remains from the moment of their prediction [12].

Since the process of light propagation in media is described by the nonlinear Schrödinger equation (NSE) or a system of NSEs, developing numerical and analytical methods for finding the solitons is an important task. As a rule, many known solitons are found analytically [2, 13]. Nevertheless, the construction of soliton solutions on the basis of computer simulations is also widely discussed. One of the possible methods to obtain the eigenfunction (EF) and eigenvalue (EV) of the NSE (or the system of such equations) consists in the application of methods described in [7, 8]. This approach is even more important for the tasks of the transmission of information via optical fibers, since the obtained EF on the transverse space coordinate are the nonlinear modes of optical waveguide [1].

In the present paper the method of finding EFs and EVs for the system of two NSEs, that was proposed in [18], is generalized to the case of variable coefficients, which characterize the nonlinear response of medium in time. The main feature of the considered method is the special iterative process. In the case of the first EF, it is shown experimentally that this iterative algorithm converges for all values of examined coefficients characterizing the nonlinearity of propagation. It is tested that these EFs are soliton solutions of the discussed problem. The obtained solitons are in agreement with the analytical representation of the well-known solitary solutions. It should also be emphasized that the constructed method allows us to find another class of soliton solutions, which possess two local maximums. Such solitons exist for a wide range of coefficients of quadratic and cubic nonlinearity as well. The reported method can be used to obtain EFs of higher, than second order. However, in that case a convergence of the iterative process strongly depends on the choice of the initial approximation.

2 Basic Equations

The dimensionless equations that describe SHG process by femtosecond pulse in optical fiber (or for a medium which length is many times smaller than the diffraction length), taking into account the pulse self-action due to a cubic nonlinear response, are the following:

$$\frac{\partial A_1}{\partial z} + iD_1\frac{\partial^2 A_1}{\partial t^2} + i\gamma(t)\ A_1^* A_2 e^{-i\Delta kz} + i\alpha(t)A_1\big(|A_1|^2 + 2\,|A_2|^2\big) = 0, \quad (2.1)$$
$$\frac{\partial A_2}{\partial z} + \nu\frac{\partial A_2}{\partial t} + iD_2\frac{\partial^2 A_2}{\partial t^2} + i\gamma(t)\ A_1^2 e^{i\Delta kz} + i2\alpha(t)A_2\big(2\,|A_1|^2 + |A_2|^2\big) = 0,$$
$$\gamma(t) = \gamma_0\big(1 + \delta_\gamma \sin(\pi n_\gamma t/L_t)\big), \quad \alpha(t) = \alpha_0\big(1 + \delta_\alpha \sin(\pi n_\alpha t/L_t)\big).$$

The problem is solved in the domain $\{0 < z \le L_z, 0 < t < L_t\}$. Here A_j are complex amplitudes of harmonics ($j = 1, 2$), normalized to the square root of the maximal intensity of optical wave in the input section of medium ($z = 0$), t is the dimensionless time in the system of coordinates that accompanies the pulse on the fundamental frequency or the transverse coordinate that is normalized to a radius of input laser beam, L_t is dimensionless time interval during which the process is analyzed, z is the normalized longitudinal coordinate, L_z is its maximum, D_j $(j = 1, 2)$ are coefficients that characterize the second order dispersion or diffraction of laser beams, $\gamma(t)$ is the coefficient of nonlinear coupling of interacting waves, $\Delta k = k_2 - 2k_1$ describes the dimensionless mismatching of their wave numbers, $\alpha(t)$ characterizes the self-action of waves due to a cubic nonlinear response. Parameter ν is proportional to the difference of the inverse values of group velocities of the second harmonic wave and the basic one if we analyze the laser pulse propagation or it characterizes birefringence of laser beams (difference between propagation directions of laser waves with different frequencies). For simplicity, we will discuss only the case of laser pulse propagation and consider the case of group velocity matching, i.e. $\nu = 0$.

We emphasize once more, that in this paper we find EFs and EVs when coefficients of nonlinear response depend on time. This situation was not discussed in [17, 18].

For equations (2.1) the following initial conditions are necessary:

$$A_j(t, z = 0) = A_{0j}(t), \quad j = 1, 2, \quad 0 \leqslant t \leqslant L_t, \tag{2.2}$$

where A_{0j} are dimensionless amplitudes normalized to $\max\limits_{t,j} |A_{0j}(t)|$. Because of the finiteness of initial distribution and limited length of medium the boundary conditions for equations (2.1) can be written as follows:

$$A_j \,|_{t=0,L_t} = 0 \,. \tag{2.3}$$

To find EFs of equations (2.1) the solution is presented as $A_1 = u(t)e^{-i\lambda z}$, $A_2 = v(t)e^{-i\mu z}$. Substituting these functions into equations (2.1), the following system of boundary value problems is derived:

$$\begin{cases} D_1 \dfrac{d^2u}{dt^2} + \gamma(t)\ u^* v e^{i(2\lambda-\mu-\Delta k)z} + \alpha(t)u\left(|u|^2 + 2|v|^2\right) = \lambda u, \\ D_2 \dfrac{d^2v}{dt^2} + \gamma(t)\ u^2 e^{-i(2\lambda-\mu-\Delta k)z} + 2\alpha(t)v\left(2|u|^2 + |v|^2\right) = \mu v \\ u(0) = u(L_t) = v(0) = v(L_t) = 0. \end{cases} \tag{2.4}$$

In order to remove the dependence of the coefficient of quadratic nonlinearity on the coordinate z, the parameter μ is set equal to $2\lambda - \Delta k$. Then equations (2.4) can be written in a simpler form:

$$\begin{aligned} &D_1 \frac{d^2u}{dt^2} + \gamma(t)\ u^* v + \alpha(t)u\left(|u|^2 + 2|v|^2\right) = \lambda u, \\ &D_2 \frac{d^2v}{dt^2} + \gamma(t)\ u^2 + 2\alpha(t)v\left(2|u|^2 + |v|^2\right) + \Delta k v = 2\lambda v. \end{aligned} \tag{2.5}$$

It is easy to prove that equations (2.5) possess only real EV. Actually, if the first equation multiplied by u^* is added to the second one multiplied by v^*

and the obtained identity is integrated from 0 to L_t, then using boundary conditions (2.3) we obtain:

$$\int_0^{L_t} \Big(-D_1\Big|\frac{du}{dt}\Big|^2 - D_2\Big|\frac{dv}{dt}\Big|^2 + 2\gamma(t)Re(u^2v^*) + \alpha(t)|u|^2\left(|u|^2+2|v|^2\right)\Big)\,dt$$
$$+\int_0^{L_t}\Big(2\alpha|v|^2\left(2|u|^2+|v|^2\right)+\Delta k|v|^2\Big)\,dt = \lambda\int_0^{L_t}\Big(|u|^2+2|v|^2\Big)\,dt.$$

Taking into account that integrals on the left and right sides of the equality are real, we derive that λ is real. However, in general case EFs in contrast to EVs can be real as well as complex. Below only real EFs are assumed.

Thus, we get the system:

$$\begin{cases} D_1\dfrac{d^2u}{dt^2} + \gamma(t)uv + \alpha(t)u\left(|u|^2+2|v|^2\right) = \lambda u, \\ \dfrac{D_2}{2}\dfrac{d^2v}{dt^2} + \dfrac{\gamma(t)}{2}u^2 + \alpha(t)v\left(2|u|^2+|v|^2\right) + \dfrac{\Delta k}{2}v = \lambda v. \end{cases} \tag{2.6}$$

3 Numerical Method

Let us introduce the uniform grid $\omega_t = \{t_j = j\tau, j = \overline{0, N_t}, L_t = \tau N_t\}$ and define the grid functions u_h, v_h, γ_h , α_h on ω_t: $u_j = u(t_j)$, $v_j = v(t_j)$, $\gamma_j = \gamma(t_j)$, $\alpha_j = \alpha(t_j)$. The difference Laplace operator is defined as $f_{\bar{t}t,j} = (f_{j+1} - 2f_j + f_{j-1})/\tau^2$.

The difference scheme for equations (2.6), approximating them with the second order on segment $[0,L_t]$, are written as:

$$D_1u_{\bar{t}t,j} + \gamma_j\ u_jv_j + \alpha_ju_j\left(u_j^2+2v_j^2\right) = \lambda u_j, \tag{3.1}$$
$$\frac{D_2}{2}v_{\bar{t}t,j} + \frac{\gamma_j}{2}\ u_j^2 + \alpha_jv_j\left(2u_j^2+v_j^2\right) + \frac{\Delta k}{2}v_j = \lambda v_j, \quad j=\overline{1, N_t-1}$$

subject to the zero boundary conditions: $u_o = u_{N_t} = v_o = v_{N_t} = 0$. Substituting $w_j = \sqrt{2}v_j$ into equations (3.1) we rewrite this problem as

$$D_1u_{\bar{t}t,j} + \frac{\gamma_j}{\sqrt{2}}\ u_jw_j + \alpha_ju_j\left(u_j^2+w_j^2\right) = \lambda u_j,$$
$$\frac{D_2}{2}w_{\bar{t}t,j} + \frac{\gamma_j}{\sqrt{2}}\ u_j^2 + \alpha_jw_j\left(2u_j^2+0.5w_j^2\right) + \frac{\Delta k}{2}w_j = \lambda w_j,\ j=\overline{1, N_t-1}.$$

Since above equations are nonlinear the following iterative process is used for $j=\overline{1, N_t-1}$ to solve them:

$$D_1 \overset{s+1}{u}_{\bar{t}t,j} + \frac{\gamma_j}{\sqrt{2}} \overset{s}{u}_j \overset{s+1}{w}_j + \alpha_j \overset{s+1}{u}_j \left(\overset{s}{u}{}_j^2 + \overset{s}{w}{}_j^2\right) = \overset{s+1}{\lambda} \overset{s+1}{u}_j,\quad s=0,1,2,\ldots, \tag{3.2}$$
$$\frac{D_2}{2} \overset{s+1}{w}_{\bar{t}t,j} + \frac{\gamma_j}{\sqrt{2}} \overset{s}{u}_j \overset{s+1}{u}_j + \alpha_j \overset{s+1}{w}_j \left(2\overset{s}{u}{}_j^2 + \frac{1}{2}\overset{s}{w}{}_j^2\right) + \frac{\Delta k}{2} \overset{s+1}{w}_j = \overset{s+1}{\lambda} \overset{s+1}{w}_j,$$

The terms, corresponding to the quadratic nonlinearity, are arranged so that, firstly, the matrix of linear system of equations (3.2) is symmetrical, and secondly, the first equation in (3.2) on iteration $(s+1)$ takes into the dependence of the solution on w, and the second equation on u.

Realization of the iterative process requires to specify initial approximations for u and w on the zero iteration $(s=0)$. We analyze two kinds of initial approximations: taking EF of the corresponding linear problem :

$$\overset{s=0}{u_m} = \overset{s=0}{v_m} = \sin\left(\frac{\pi m t}{L_t}\right), \quad m = 1, 2, \ldots. \tag{3.3}$$

and Gaussian distribution:

$$\overset{s=0}{u_m} = \overset{s=0}{v_m} = \exp\left[-(t - L_t/2)^2/\tau_p^2\right]. \tag{3.4}$$

Here τ_p is a positive parameter. The selection of an initial approximation can influence the form of the obtained EFs for certain numbers of EVs. It should be stressed that for finding the first EF the selection of the initial approximation is not important. But for obtaining the second EF the choice of initial approximation is essential and it defines the form of obtained solitons. If one takes an initial approximation in the form of the exact second EF of the linear problem described by (3.3), $m = 2$, then the solution will be the composition of two first EFs, located far enough from each other. This case is trivial one and will not be considered by us. Choosing initial approximation (3.4) for obtaining second EF gives essentially different solution.

Introducing vector $\psi = (u_1, w_1, u_2, w_2, \ldots, u_{N_t-1}, w_{N_t-1})$ we write equations (3.2) in the matrix form:

$$\overset{s}{\Lambda}\,\overset{s+1}{\psi} = \overset{s+1}{\lambda}\,\overset{s+1}{\psi},$$

where $\overset{s}{\Lambda}$ is a real symmetrical five-diagonal matrix:

$$\begin{pmatrix}
\alpha_1 \overset{s}{a_1} - \dfrac{2D_1}{\tau^2} & \dfrac{\gamma_1}{\sqrt{2}}\overset{s}{u_1} & \dfrac{D_1}{\tau^2} & 0 & 0 & 0 \\
\dfrac{\gamma_1}{\sqrt{2}}\overset{s}{u_1} & \alpha_1 \overset{s}{b_1} + \dfrac{\Delta k}{2} - \dfrac{D_2}{\tau^2} & 0 & \dfrac{D_2}{2\tau^2} & 0 & 0 \\
\dfrac{D_1}{\tau^2} & 0 & \alpha_2 \overset{s}{a_2} - \dfrac{2D_1}{\tau^2} & \dfrac{\gamma_2}{\sqrt{2}}\overset{s}{u_2} & \dfrac{D_1}{\tau^2} & 0 \\
0 & \dfrac{D_2}{2\tau^2} & \dfrac{\gamma_2}{\sqrt{2}}\overset{s}{u_2} & \alpha_2 \overset{s}{b_2} + \dfrac{\Delta k}{2} - \dfrac{D_2}{\tau^2} & 0 & \dfrac{D_2}{2\tau^2} \\
\cdots & \cdots & \cdots & \cdots & \cdots & \cdots
\end{pmatrix},$$

$$\overset{s}{a_j} = \overset{s}{u_j^2} + \overset{s}{w_j^2}, \quad \overset{s}{b_j} = 2\,\overset{s}{u_j^2} + 0.5\,\overset{s}{w_j^2}.$$

Vector $\overset{s}{\psi}$ is normalized according to the condition $\max\limits_j |\,\overset{s}{\psi_j}\,| = 1$. The iterative process is completed, if the stopping criterion is reached:

$$\left|\overset{s+1}{\lambda} - \overset{s}{\lambda}\right| < \varepsilon|\,\overset{s}{\lambda}\,| + \delta, \quad \varepsilon, \delta > 0. \tag{3.5}$$

As it can be seen, the original problem of obtaining the functions $u(t), v(t)$ is reduced to searching EFs and EVs of the matrix $\overset{s}{\Lambda}$. For this purpose at each iteration the matrix is reduced to the three-diagonal form by using the method of plane rotations [7, 8]. EVs of a real symmetrical three-diagonal matrix are obtained using the QL algorithm. After this, the EF corresponding to EV is calculated by the method of inverse iterations [7, 8]. The obtained EF is used to construct the matrix $\overset{s+1}{\Lambda}$ in the next iterative step. This process is repeated, until condition (3.5) is satisfied. We note that choosing of EF corresponding to EV is made like to [18]. It is very important to notice that for constant coefficients, which characterize the process of nonlinear propagation, the iterative process mentioned above allows to find EF and EV for any coefficients of nonlinearity under consideration. The convergence of the presented algorithm was checked numerically for a wide range of parameters characterizing the nonlinearity of medium. Only the validity of a unique condition with regard to dependencies of coefficients in (2.1) is required for the convergence of the proposed iterative procedure.

4 Soliton Solutions

As the illustration of the presented algorithm we show the EF for the first EV (see Fig. 1) in the case which is close to the physical experiment [3]. The following values of parameters were used: $D_1 = 0.04$, $D_2 = 0.14$, $\alpha_0 = 5$, $\gamma_0 = 20$, $\Delta k = 0$. This set of parameters is used in all experiments presented in this paper. To make sure that obtained EFs are optical solitons, distributions of amplitudes, were used as the initial conditions for equations (2.1). To solve (2.1) we have used a nonlinear conservative finite-difference scheme.

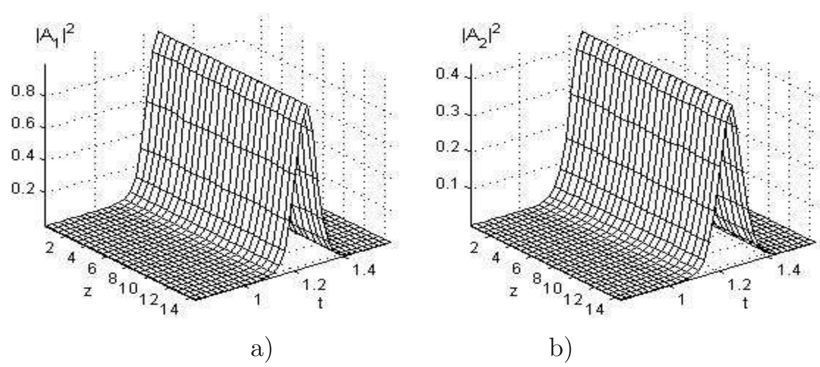

Figure 1. Evolution of solitary waves on fundamental (a) and doubled (b) frequency corresponding to the first eigenvalue for the following set of parameters: $\delta_\alpha = 0, \delta_\gamma = 1, n_\gamma = 1, L_z = 15$.

Since the dimensionless coefficients of the second order dispersion are less than 1, the soliton propagation was analyzed for a sufficiently large medium length $0 \leqslant z \leqslant 15$. The discrete steps on time and space were chosen equal to

0.0025 and 0.001 correspondingly. One can easily see from Fig. 1 that the peak intensity of harmonics does not change along the propagation direction and the shapes of pulses also remain constant. It is also important to emphasize that EFs of original problem (2.1), corresponding to the first EV, do not change with an increase of time domain, thus, they are optical solitons. These solitons are stable with respect to perturbations of their initial form that can reach 20% of their amplitudes. Such perturbations lead to the periodic oscillations of the peak intensity of both harmonics along their propagation. We would also like to stress that the form of the soliton solutions, which correspond to first EV, does not depend on the choice of initial approximation for iterative process (3.2).

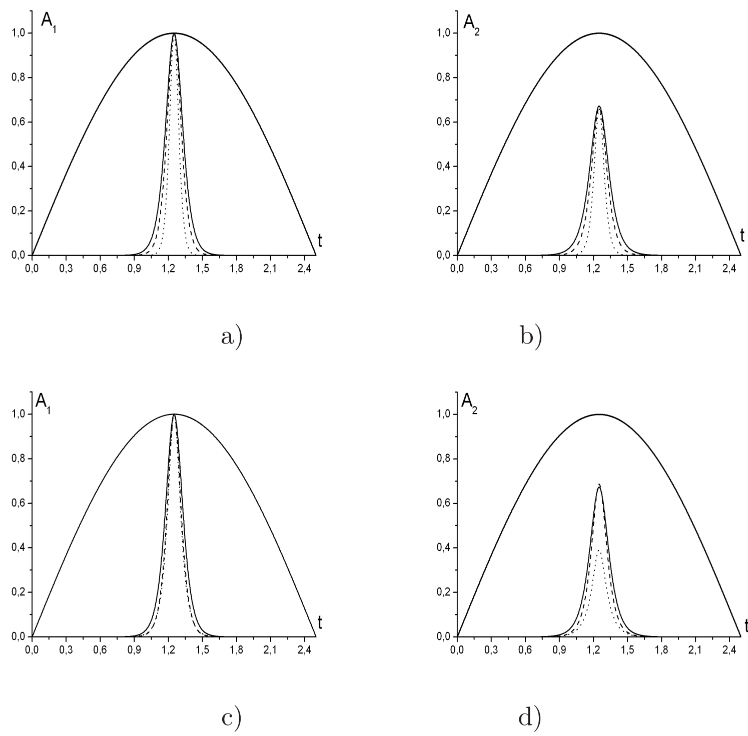

Figure 2. Shape of pulse for solitary wave on fundamental (a,c) and doubled (b,d) frequency corresponding to the first eigenvalue under the fixed value $\delta_\alpha = 0$ (a,b) for $n_\gamma = 1$, $\delta_\gamma = 0$(solid line), $\delta_\gamma = 1$(dashed line), $\delta_\gamma = 5$(dotted line) or under the fixed value $\delta_\gamma = 0$ (c, d) for $n_\alpha = 1$, $\delta_\alpha = 0$ (solid line), $\delta_\alpha = 1$ (dashed line), $\delta_\alpha = 5$ (dotted line).

We have also investigated how the non-uniformity of nonlinear coefficients affects the soliton shape. The results shown on Fig. 2 – Fig. 4 were obtained by using the described above algorithm for various values of δ_α and δ_γ. On the base of these pictures and the other computer simulation results we make the following conclusions. For even values n_γ or n_α our iterative algorithm of finding EVs and EFs does not converge. The reason is absence of the first derivative in time for functions in equation (2.4), since the solution in this case is unsymmetrical and the presence of the first derivative in equation (2.4) is a necessary condition.

For $n_\gamma = 1$ and $n_\alpha = 1$ (see Fig. 2) with increasing of δ_γ or δ_α the pulse duration on both frequencies is decreasing. Nevertheless, the increasing of δ_γ influences more strongly on pulse duration than increasing of δ_α.

If modulation of coefficients of nonlinearity takes place with $n_\gamma = 3$ and $n_\alpha = 3$ (see Fig.3) then an increase of δ_α results in broadening of pulses duration on both frequencies. The influence of perturbation of nonlinear coefficient γ depends on its amplitude. If the amplitude is less than 2 then the dependence mentioned above takes place. But for an amplitude of perturbation, which is greater than 2, an increasing in the perturbation amplitude results in decreasing of pulses duration.

The further increasing of frequency of perturbation of nonlinear coefficients $n_\gamma = 5$ and $n_\alpha = 5$ (see Fig.4) gives the same dependencies between pulses duration and amplitudes of perturbations as for $n_\gamma = 1$ and $n_\alpha = 1$.

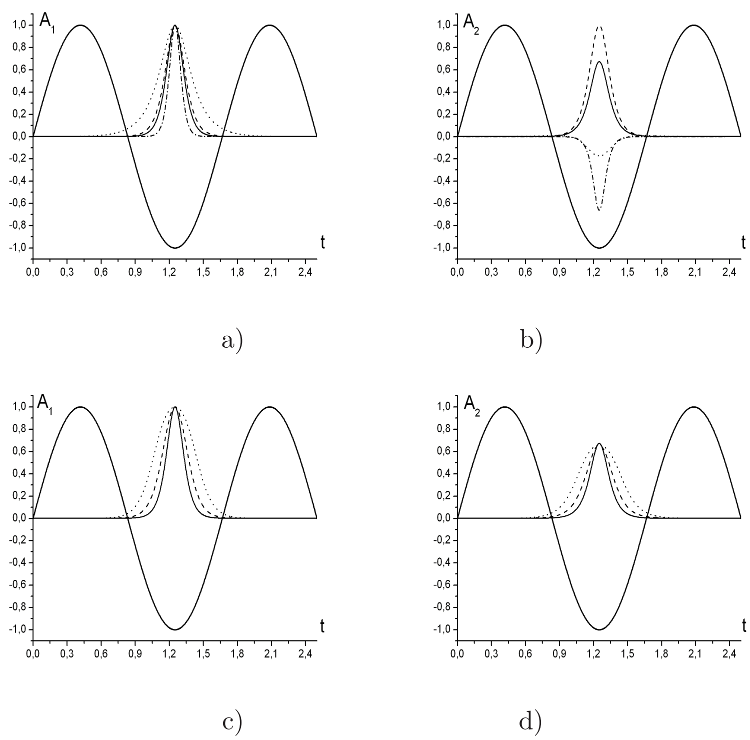

Figure 3. Shape of pulse for solitary wave on fundamental (a,c) and doubled (b,d) frequency corresponding to the first eigenvalue under the fixed value $\delta_\alpha = 0$ (a,b) for $n_\gamma = 3$, $\delta_\gamma = 0$ (solid line), $\delta_\gamma = 0.5$ (dashed line), $\delta_\gamma = 1$ (dotted line), $\delta_\gamma = 4.5$ (dash-dotted line) or under the fixed value $\delta_\gamma = 0$ (c, d) for $n_\alpha = 3$, $\delta_\alpha = 0$ (solid line), $\delta_\alpha = 1$ (dashed line), $\delta_\alpha = 1.5$ (dotted line).

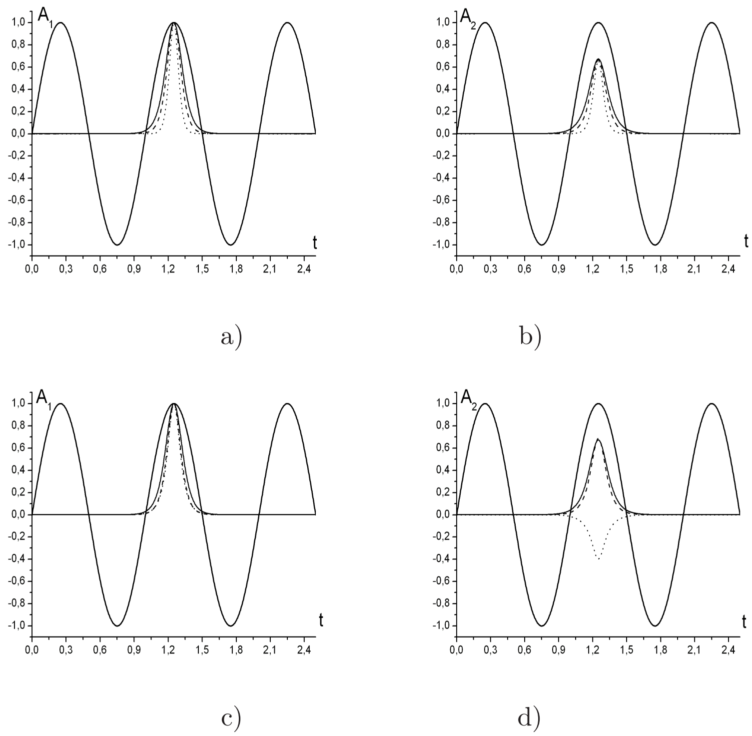

Figure 4. Shape of pulse for solitary wave on fundamental (a,c) and doubled (b,d) frequency corresponding to the first eigenvalue under the fixed value $\delta_\alpha = 0$ (a,b) for $n_\gamma = 5$, $\delta_\gamma = 0$ (solid line), $\delta_\gamma = 1$ (dashed line), $\delta_\gamma = 5$ (dotted line) or under the fixed value $\delta_\gamma = 0$ (c, d) for $n_\alpha = 5$, $\delta_\alpha = 0$ (solid line), $\delta_\alpha = 1$ (dashed line), $\delta_\alpha = 5$ (dotted line).

It should be noticed that proposed algorithm allows to obtain EFs, corresponding to the second EV as well. In contrast to the first EV, their form depends on initial approximation. When initial approximation is taken in the form (3.3), the solution is the composition of the already obtained solitons for first EV, located on a big enough distance from each other. It is obvious that solutions of such type will also be solitons. This case is trivial one. EFs with essential another shape were obtained with the choice of initial approximation in the form of Gaussian distributions (3.4) with $\tau_p = 1$. In this case the soliton has a shape with a local minimum at its center [18]. The form of EFs for second EV is shown on Fig. 5.

If these EFs are used as initial conditions for system (2.1), optical waves propagate without any changes. Increasing of the time domain by adding zeros to EF at the ends of the time interval does not affect the evolution of intensities of both harmonics. This fact proves that obtained EFs are also optical solitons. However, in contrast to the solitons, which correspond to the first EV, this soliton is unstable to small perturbations, which are greater than 5% of its value.

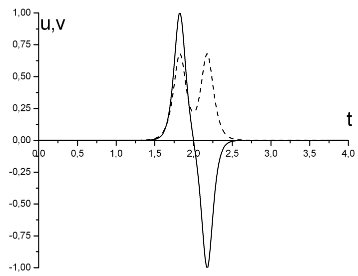

Figure 5. Shape of pulse for solitary wave on fundamental (solid line) and doubled (dashed line) frequency corresponding to the second eigenvalue for the following set of parameters: $\delta_\alpha = 0$, $n_\alpha = 0$, $\delta_\gamma = 0$, $n_\gamma = 0$.

5 Conclusion

The iterative method for finding solitons, that was proposed in [18], can be generalized as well under the condition of dependence of coefficients of nonlinearity on time or transverse coordinate for the system of two Schrödinger equations with the combined nonlinearity. These solitons, corresponding to the first EV, are stable with respect to initial perturbations of their forms. The developed method is also applicable for finding EFs of higher order. However, its convergence (just as in [18]) depends on the choice of initial approximation of EF. At the same time, the coefficient of cubic nonlinearity plays special role. It should be noticed that with an increase of time interval the EVs tend to approach the value $\alpha/2$. To find a soliton in this case it is necessary either to decrease the time interval or to use a grid with the more coarse step to separate EVs from each other.

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