



## POWER PLANT INVESTMENT PLANNING BY STOCHASTIC PROGRAMMING

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**Abstract.** Although the problem of rational power generation has been extensively studied, traditional approaches for power optimization do not offer good solutions to this purpose, especially in a competitive electricity market environment where many factors are uncertain. In this paper, within the framework of two-stage linear stochastic programming, the method for power planning has been developed, with uncertain factors taken into account, through a continuously distributed set of scenarios. The objective is to find the structure of the power plants capacity in the region which minimizes the sum of the investment and the expected operating costs over the long-term planning horizon, taking into account the environmental impact. The structure of the considered task corresponds to a power investment planning problem that often arises in the developing regions. The method is developed for solving the stochastic optimization problem by the sequence of Monte-Carlo sampling estimators. The procedures developed make it possible to solve stochastic problems with an admissible accuracy by means of an acceptable amount of computations. As follows from numerical experiments the approach presented enables us to decrease the total expected costs of power planning versus deterministic planning solution.

**Keywords:** stochastic programming, Monte Carlo method, power planning, stochastic gradient, statistical criteria,  $\epsilon$ -feasible direction.

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### 1. Introduction

The energy sector needs to be environmentally sustainable while as being economically sustainable. Energy utilities need to earn an adequate return and satisfy shareholders, whilst understanding their corporate responsibilities and a wider social impact of their business. Uncertainty of the power market should be taken into account when planning the power system (Freund 2004; Beraldi *et al.* 2008; Fleten and Kristoffersen 2008). Although the prob-

lem of rational power generation under uncertainty has been extensively studied, traditional planning models and methods do not offer good solutions to this problem, especially in a competitive electricity market environment where many factors are uncertain (Graymore *et al.* 2008; Grundey 2008). In this paper the problem is considered of an energetic concern that has to invest in a regional system of power plants to meet current and future demand for electrical power of the region. These plants are to be built for the first year only, and are expected to operate for some years under a certain budget, which is to be allocated for different types of plants. A stochastic optimization model is formulated under the presumption that the generation outputs and load demands can be modelled as following to specified continuous probability distributions. The objective is to find the structure of the power plant capacity that minimizes the sum of the investment cost and the expected value of the operating cost over the planning horizon taking into account the impact on the environment including the appropriated environmental costs to operating ones. The optimization problem is solved by means of a novel numerical approach that exploits a particular problem structure. Finally, we report some preliminary computational experiments.

## 2. Power Plant Investment Planning Problem

The structure of the task considered corresponds to the power investment planning problem that often arises in the developing regions. The model and data considered in the paper involve main parts of the sustainable energetic development of the region in the long term perspectives and are created following to sources (Freund 2004; Hezri and Dovers 2006; Karger and Hennings 2009; Paiders 2008; Stepanonytė and Blynas 2008; Training Material in Financial Engineering 2009).

Let the plants to be projected for the open field investment. Say, the details of the problem to be solved are as follows. Let the plants be expected to operate over  $T = 15$  years. The budget for the construction of power plants is  $S = \$10$  billion, which is to be allocated for  $n = 4$  different types of plants: gas turbine, coal, nuclear power, and hydroelectric. The budget should take into account the discounting factor, because usually investments in construction and building of power plants are not done once. The objective is to minimize the sum of the investment cost and the expected value of the operating cost over  $T$  years that leads to minimizing the influence upon the nature during the process of power generation. Assume, the prognosticated demand for electric power is distributed with a specified probability distribution. Assume,  $m=5$  blocks of demand are requiring for different amount of power during the year.

Known solutions of power planning under uncertainty exploit the two-stage or multi-stage stochastic programming framework with a discrete number of scenarios (see Ruszczyński and Shapiro 2003; Wallace and Fleten 2003; Beraldi *et al.* 2008; Fleten and Kristoffersen 2008). However, the adequate evaluation of scenarios and its probabilities is not an easy task in practice, therefore the analysis of a continuously distributed set of scenarios may offer more opportunities. The parameters of distribution of the scenario are choosing by means of statistical analyses of historical data of energetic development of the region. Thus, without loss of generality assume the demands in blocks to be independently and normally distributed.

The mean and standard deviations of the expected power demands that with durations of the blocks are shown in Table 1.

Power plants are priced according to their electric production capacity, measured in gigawatts (GW). Table 2 shows the investment cost for each type of plants (Freund 2004).

**Table 1.** Power demand during the year

Demand Block	Expected power demand, $\mu$ (GW)	The standard deviation of power demands, $\sigma$ (GW)	Block duration, (h)
1	26.0	1.3	490
2	21.5	1.1	730
3	17.3	0.9	2190
4	13.9	0.7	3260
5	11.1	0.6	2090

**Table 2.** Investment costs for power plants

Power plant type	Cost, \$ $10^8$ /GW
Gas Turbine	1.1
Coal	1.8
Nuclear power	4.5
Hydroelectric	9.5

Since the production of hydroelectric energy depends on the availability of rivers that may be dammed, the geography of the region constrains the hydroelectric power capacity no more than  $P = 5.0$  GW.

The operating costs for the first year of each type of power plants, as well as the cost of purchasing power from an external source, are shown in Table 3, where the units are in cents per kilowatt-hour (KWh). The operating costs consist of the expenses for expenditures of fuel, water and extra electric power, chemical and technological materials and of the taxes for air and water pollution, the discounting factor, the costs of utilization of waste products and of the activity for the nature safety (Hezri and Dovers 2006; Karger and Hennings 2009; Paiders 2008; Training Material in Financial Engineering 2009). These expenses depend linearly on the capacity of the plants to be built (Stepanonytė and Blynas 2008; Training Material in Financial Engineering 2009). The environmental issues of power plant construction and exploitation are related with waste and water usage. The waste products of the gas turbine plant are CO, CO<sub>2</sub>, NO<sub>x</sub>, while solid particles and SO<sub>2</sub> are additional waste products of the coal plant. The factors of emissions of coal and gas turbine plants are shown in Table 4.

**Table 3.** Power generation operating costs

Power plant type	Operating cost, (€/kWh)
Gas Turbine	3.92
Coal	2.44
Nuclear	1.40
Hydroelectric	0.40
External Source	15.0

**Table 4.** Factor of the emissions of natural gas and coal

Factor of the emissions	Natural gas*	Coal**
SO <sub>2</sub>	0.10	16.0
NO <sub>x</sub>	1.00	4.5
CO	0.01	0.03
Volatile organics	0.01	0.8
Solid particles	0.001	1.4
CO <sub>2</sub> (t/T)	56.9	94.6

\* g/kg, \*\* l/kg.

The amount of water usage is often of great concern for electricity generating systems as populations increase and droughts become a concern. General numbers for fresh water usage of different power sources are shown in Table 5.

**Table 5.** Fresh water usage for different power sources

Power source	Water usage (m <sup>3</sup> /GWh)
Natural gas	150
Coal	480
Nuclear power	550
Hydroelectric	1430

The operating costs for hydroelectric and nuclear plants are lower as these plants make smaller influence upon the natural environment. The operating costs for nuclear plants don't estimate the costs for the utilization of the waste products after closing the plant but corresponding data might be easily included to the task.

Thus, the problem is modeled as a two-stage stochastic linear program (SLP) model

$$\sum_{i=1}^n c_i x_i + E \left( \min_{y \geq 0} \left( \sum_{i=1}^{n+1} \sum_{j=1}^m \sum_{k=1}^T q_i h_j y_{ijk} \right) \right) \rightarrow \min_{x \geq 0} \quad (1)$$

subject to

$$\begin{aligned}
 & \sum_{i=1}^n c_i x_i \leq S, \\
 & x_4 \leq P, \\
 & y_{ijk} \leq x_i, i = 1, 2, 3, 4, \forall j, k, \\
 & \sum_{i=1}^{n+1} y_{ijk} \geq D_{jk}, \forall j, k, \\
 & \sum_{i=1}^{n+1} y_{ijk} \leq w_k, \forall j, k, \\
 & x \geq 0, y \geq 0,
 \end{aligned} \tag{2}$$

where

$S$  is the budget for the construction,

$P$  is the capacity of the hydroelectric power,

$T$  is the number of years,

$n$  is the number of the different types of power plants,

$m$  is the number blocks of demand,

$x = (x_1, x_2, x_3, x_4)$  is a vector, representing the capacity to be built for each type of plant,

$y_{ijk}$  is the amount of electricity capacity used to produce electricity by power plant type  $i$  for demand block  $j$  in year  $k$  in GW,

$c_i$  is the investment cost per GW of capacity for power plant type  $i$ ,

$q_i$  is the operating cost of power generation for power plant type  $i$ ,

$h_j$  is the duration of the demand block  $j$ ,

$D_{jk}$  is the power demand in year  $k$  at demand block  $j$ :  $N(\mu_p, \sigma_j)$ ,

$w_k$  is the limitation of the fresh water usage in year  $k$ .

Since the model aims to find the best structure of power plants capacity to satisfy the region power demands the vector  $x$  is assumed to be varying continuously. Of course, the values of the capacities should be specified implementing the solution, while the details of facility allocation and possible technical solutions are taken into account.

### 3. Monte-Carlo estimators for stochastic optimization

The adaptive method for solving SLP (1), (2) by series of Monte-Carlo samples is applied, exploiting the asymptotic properties of Monte-Carlo sampling estimators. This method is based on the handling of a statistical simulation error in a statistical manner and the rule for iterative control of the size of Monte-Carlo samples (Sakalauskas 2002, 2004).

Details of the method are as follows, in general. Let a two-stage SLP problem with a complete recourse be considered:

$$F(x) = c \cdot x + E(Q(x, \xi)) \rightarrow \min_{x \in D \subset \mathfrak{R}_+^n} \tag{3}$$

subject to a feasible set

$$D = \left\{ x \mid A \cdot x = b, x \in \mathfrak{R}_+^n \right\}, \tag{4}$$

where

$$Q(x, \xi) = \min_y \left( q \cdot y \mid W \cdot y + T \cdot x \leq h, y \in \mathfrak{R}_+^m \right) \tag{5}$$

(See details in Sakalauskas and Zilinskas 2009).

We have by the duality that the gradient of the objective function might be expressed as

$$\nabla_x F(x) = E \left( g(x, \xi) \right), \tag{6}$$

where  $g(x, \xi) = c - T \cdot u^*$  is given by a set of solutions of a dual problem (Sakalauskas and Zilinskas 2009)

$$(h - T \cdot x)^T \cdot u^* = \max_u [(h - T \cdot x)^T \cdot u \mid u \cdot W^T + q \geq 0, u \in \mathfrak{R}^m]. \tag{7}$$

Assume that Monte-Carlo samples  $Y = (y^1, y^2, \dots, y^N)$ , where  $y^i$  are independent random variables identically distributed at density  $p(x, \cdot)$ , are provided for any  $x \in D$ . Then the sampling objective estimator

$$\tilde{F}(x) = \frac{1}{N} \sum_{j=1}^N f(x, y^j) \tag{8}$$

and the sampling variance estimator

$$\tilde{d}^2(x) = \frac{1}{N} \sum_{j=1}^N \left( f(x, y^j) - \tilde{F}(x) \right)^2 \tag{9}$$

can be obtained from these samples. Next, the gradient can be evaluated using the same random sample:

$$\tilde{G}(x) = \frac{1}{N} \sum_{j=1}^N g(x, y^j) \tag{10}$$

for any  $x \in D$  (Sakalauskas and Zilinskas 2006, 2009). The sampling covariance matrix

$$Z(x) = \frac{1}{N-n} \sum_{j=1}^N \left( g(x, y^j) - \tilde{G} \right) \cdot \left( g(x, y^j) - \tilde{G} \right)^T \tag{11}$$

is introduced, too.

#### 4. Stochastic procedure for optimization

To create an optimizing sequence, the gradient search approach with projection to a  $\epsilon$ -feasible set is applied to avoid problems of “jamming” or “zigzagging” (Sakalauskas and Zilinskas 2009). The set of *feasible directions* is defined as follows:

$$V(x) = \left\{ g \in \mathfrak{R}^n \mid Ag = 0, \forall_{1 \leq i \leq n} \left( g_i \leq 0, \text{ if } x_j = 0 \right) \right\},$$

where  $g_U$  is assumed as projection of vector  $g$  onto the set  $U$  and the  $\epsilon$ -feasible set is also defined by similar way (Sakalauskas 2004):

$$V_\epsilon(x) = \left\{ g \in \mathfrak{R}^n \mid Ag = 0, \forall_{1 \leq i \leq n} \left( g_i \leq 0, \text{ if } \left( 0 \leq x_j \leq \epsilon_x(g) \right) \right) \right\}.$$

The stochastic optimization procedure is defined in a recurrent manner:

$$x^{t+1} = x^t - \rho^t \cdot \tilde{G}_\epsilon(x^t), \tag{12}$$

where  $\tilde{G}_\varepsilon^t$  is a projection of stochastic gradient estimator (10) to the  $\varepsilon$ -feasible set,  $\rho^t = \rho_{x^t}(\tilde{G}_\varepsilon^t)$  is a step-length multiplier taken at the point  $x^t$ , and  $x^0 \in D$  is some initial point (Sakalauskas and Zilinskas 2006). Let the initial sample be generated of size  $N^0$ . Note that is no great necessity to compute estimators with a high accuracy when starting the optimization process, because then it suffices only to approximately evaluate the direction leading to the optimum. Therefore, one can obtain not so large samples at the beginning of the optimum search and, later on, increase the size of samples so as to get the estimate of the objective function with a desired accuracy just at the time of decision making on finding the solution to the optimisation problem. Thus, the following rule is proposed for regulating the sample size (Sakalauskas 2004):

$$N^{t+1} = \min \left( \max \left( \left[ \frac{n \cdot \text{Fish}(\gamma, n, N^t - n)}{\rho^t \cdot (\tilde{G}(x^t)) \cdot (Z(x^t))^{-1} \cdot (\tilde{G}(x^t))} \right] + n, N_{\min} \right), N_{\max} \right). \tag{13}$$

Minimal  $N_{\min}$  (usually  $\sim 100$ ) and maximal  $N_{\max}$  (usually  $\sim 1\,000\,000$ ) values are introduced to avoid great fluctuations of sample size in iterations.  $N_{\max}$  can also be chosen from the conditions on the permissible confidence interval of estimates of the objective function.

A possible decision on finding of optimal solution should be examined at each step of the optimization process. Since we know only the Monte-Carlo estimates of the objective function and that of its gradient, we can test only the statistical optimality hypothesis. As far as the stochastic error of these estimates depends in essence on the Monte-Carlo samples size, a possible optimal decision could be made, if, first, there is no reason to reject the hypothesis of equality to zero of the gradient, and, second, the sample size is sufficient to estimate the objective function with the desired accuracy. The following criteria used for the making of decision on the optimal solution finding and the termination of the algorithm:

1. The optimality hypothesis is accepted for some point  $x_t$  with the significance  $1 - \mu$ , if

$$\frac{1}{n} \cdot (N^t - n) \cdot \tilde{G}(x^t) \cdot (Z(x^t))^{-1} \cdot \tilde{G}(x^t) \leq \text{Fish}(\mu, n, N^t - n). \tag{14}$$

2. The objective function is estimated with permissible accuracy  $\delta$ , if

$$\frac{2\eta_\beta \cdot \tilde{D}(x^t)}{\sqrt{N^t}} \leq \delta, \tag{15}$$

where  $\tilde{D}(x^t)$  is a sampling variance of the objective function,  $\eta_\beta$  is the  $\beta$  – quantile of a standard normal distribution (Sakalauskas 2002, 2004).

The procedure (12) is iterated adjusting the sample size according to (13) and testing conditions (14) and (15) at each iteration. If the latter conditions are met both at some iteration, then there are no reasons to reject the hypothesis on the optimum finding. Therefore, there is a basis to terminate the optimization and make a decision on the optimum finding with an admissible accuracy. If at least one condition is unsatisfied, then the next sample is generated and the optimization is continued. The convergence analysis shows that optimization process should terminate after generating a finite number of Monte-Carlo samples (Sakalauskas 2002).

## 5. Implementation in power investment planning

The approach developed is implemented to solve the two-stage SLP problem of power investment planning defined above. The limit for the fresh water usage is  $5 \cdot 10^4 \text{ m}^3$  per year ( $w_k = 5 \cdot 10^4, k = 1, 2, \dots, 15$ ).

Thus, the first stage contains 6 variables and 4 restrictions, while the second stage contains 375 variables and 375 restrictions. The solving algorithm was terminated after 123 iterations. The size of the last Monte-Carlo sample is 15887, whereas the size of all Monte-Carlo samples is 290932. Hence, the method requires only 18.3 times (ratio) more computations in total than the calculation of one function value. Details of the computational experiment performed are reported in Figs 1–4 to illustrate the behavior of the optimization process, where the dependencies of the objective function, the sample size, the confidence interval of the objective function and the Hotelling statistic by the iteration number  $t$  are given.

The optimal cost of power plant investment planning problem is  $\$16.508 \pm 0.028$  billion versus the deterministic cost  $\$17.137 \pm 0.053$  billion which illustrates the importance of uncertainty to be taken into account when investments are planned. Traditional planning methods are using deterministic approach to describe the demand. Thus the deterministic problem was constructed solving the problem (1), (2) while the demand of the electric power is equal to the expected demand  $\mu_j$  ( $\sigma_j = 0, j = 1, 2, \dots, 5$ ).

The problem solution is shown in Table 6. The structure of the optimal construction decision based on stochastic data shows that no power is taken from the external sources

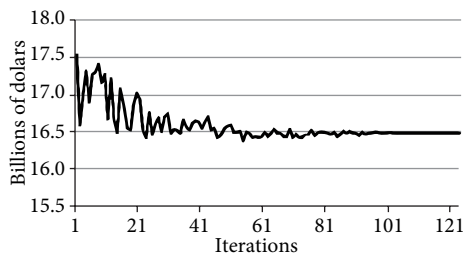


Fig. 1. Change of the objective function

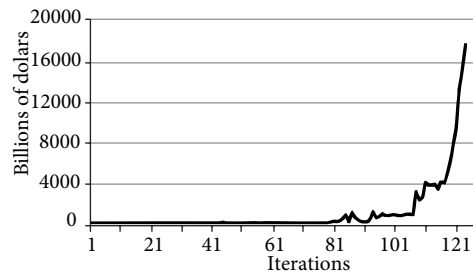


Fig. 2. Change of the sample size

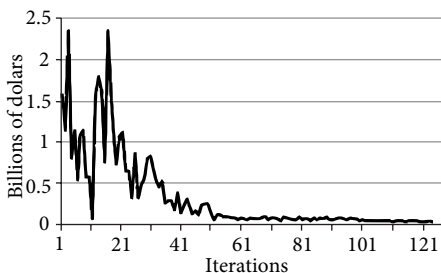


Fig. 3. Change of confidence interval

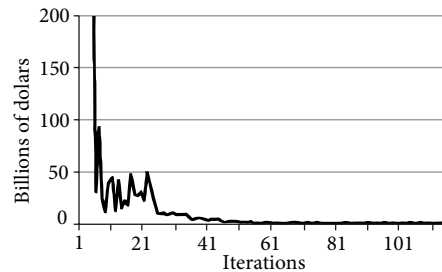


Fig. 4. Change of Hotelling statistics



and most of the electric power is generated in hydroelectric and nuclear plants which the operating costs for environmental safety are lower.

The approach developed enables us also to investigate the impact of closure costs of the nuclear power plant on profitability of a power generation system. Namely, in the fourth column of Table 6, the optimal decision without nuclear power plants is given, to which the cost of the power plant investment planning problem  $\$19.725 \pm 0.037$  billion corresponds. Thus, if the closure costs exceed about  $\$3.7$  billion, there is no reason to build the nuclear power plants.

The results of numerical experiments with various limits on the fresh water usage per year are shown in Table 7.

The changes of optimal construction of power plant capacity are given in Fig. 5 to a considerable extent by the limits of the fresh water usage for energy production.

The number of iterations, the total number of the Monte Carlo trials used for solution finding as well as the ratio of the total number of trials to number of trials taken at the last iteration are shown in Table 8 (varying the limits for the fresh water usage).

Hence, the optimization process requires only several times more computations in the whole as compared with the computation of one function value at the last iteration. The numerical experiments corroborate the theoretical conclusions on the convergence the method (Sakalauskas 2002, 2004) and shows that the approach developed makes it possible to solve stochastic power plant investment problems with an admissible accuracy by means of an acceptable amount of computations.

**Table 6.** Power plant capacity optimal construction decisions

Power plant type	Decision based on expected data (GW)	Decision based on stochastic data (GW)	Decision based on stochastic data without nuclear power plant (GW)
Gas Turbine	1.78	4.45	9.31
Coal	3.23	4.36	10.87
Nuclear	4.07	4.60	0
Hydroelectric	5.00	5.00	5.00
Cost (billion \$)	$17.137 \pm 0.053$	$16.508 \pm 0.028$	$19.725 \pm 0.037$

**Table 7.** Power plant capacity optimal construction, costs and decisions with respect to fresh water usage

Fresh water usage ( $10^4 \text{ m}^3$ )	Gas Turbine (GW)	Coal (GW)	Nuclear (GW)	Hydroelectric (GW)	Optimal cost (billion \$)
5	4.5	4.4	4.6	5.0	16.508
4	6.9	1.9	6.5	3.1	17.854
3	6.9	1.9	8.8	0.8	19.705
2	8.8	0	9.6	0	24.113
1	15.4	0	3.1	0	38.911

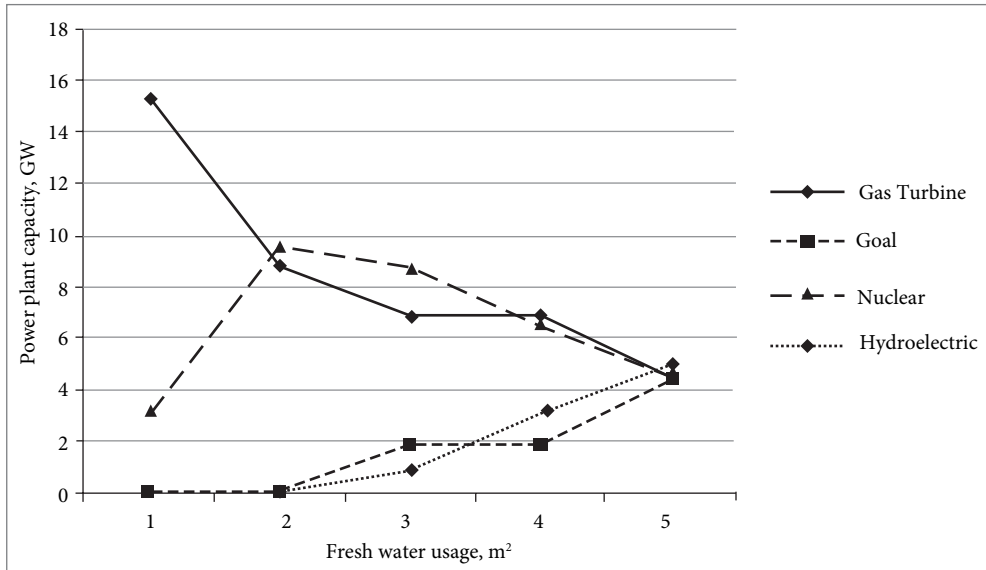


Fig. 5. Change of the power plant capacity optimal construction by fresh water usage

Table 8. Number of iterations, the total number of Monte Carlo trials used for optimal solution finding, the ratio of the total number of trials to the number of trials taken at the last iteration by the limit on the fresh water usage

Fresh water usage (10 <sup>4</sup> m <sup>3</sup> )	Confidence interval (billion \$)	Number of iterations	Total number of trials	Ratio
5	0.028	123	290932	18.3
4	0.027	127	291224	18.4
3	0.033	119	289745	18.2
2	0.034	133	292335	18.1
1	0.035	135	293457	18.5

## 6. Conclusions

The energetic planning problem taking into account the environmental impact has been considered containing the energetic situation of the region. The model and data of the problem solved involve main parts of the sustainable regional energetic development in the long term perspectives.

The main data consists of investment costs, operational generating costs with assumption of the ecological costs and energetic demand. The dynamics of demand depends on the need of the energy for particular periods of the year and is described by continuous distribution of scenarios. The generating of the power is related with large ecological costs: the expenses for expenditures of fuel, water and extra electric power, chemical and technological materials

and of the taxes for air and water pollution, the costs of utilization of waste products and of the activity for the nature safety.

The analysis of the problem in the paper indicates that effective solution can be done by stochastic programming methods. The stochastic iterative method has been developed to solve SLP problems by a finite sequence of Monte-Carlo sampling estimators applied to solve the stochastic power planning problem. The approach developed enables us to decrease the total expected costs of building and operating of regional power system versus the deterministic planning solution taking into account the impact on the environment as well as to evaluate the closure costs of nuclear plant for which a building of the nuclear power plant is profitable.

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## STOCHASTINIO PROGRAMAVIMO NAUDOJIMAS PLANUOJANT ELEKTROS ENERGIJOS INFRASTRUKTŪRĄ IR GAMYBĄ

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**Santrauka.** Nors elektros energijos infrastruktūros ir gamybos uždavinys sprendžiamas intensyviai, tradiciniai optimizavimo metodai nepateikia tinkamų sprendinių, ypač kai elektros energijos rinkoje daugelis veiksnių yra neapibrėžti. Šiame straipsnyje pateikiamas elektros energijos gamybos planavimo metodas, sukurtas remiantis dviejų etapų stochastiniu programavimu, kai neapibrėžtumas aprašomas tolydžiaisiais pasiskirstymo scenarijais. Uždavinio tikslas – rasti tinkamą regiono elektros jėgainių struktūrą, kuri minimizuotų investavimo ir ilgalaikės energijos gamybos sąnaudas. Sprendžiant uždavinį atsižvelgiama į gamtosaugos problemas. Taikant optimizavimo metodą naudojami baigtinių Monte Karlo sekų įverčiai. Siūloma procedūra leidžia išspręsti stochastinius uždavinius gana tiksliai, naudojant priimtinius skaičiavimo išteklius. Skaitiniai eksperimentai rodo, kad siūlomas metodas padeda sumažinti bendrąsias elektros energijos gamybos sąnaudas, palyginti su deterministiniu uždavinio sprendiniu.

**Reikšminiai žodžiai:** stochastinis programavimas, Monte Karlo metodas, elektros energijos gamyba, stochastinis gradientas, statistiniai kriterijai,  $\epsilon$  leistinoji kryptis.

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