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MULTI-ATTRIBUTE DECISION-MAKING METHOD RESEARCH BASED ON INTERVAL VAGUE SET AND TOPSIS METHOD

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Abstract. This paper proposed a method to resolve the multi-attribute decision-making problem using TOPSIS method based on attribute weights and attribute values are all interval vague value. Firstly, based on the operation rules of the interval Vague value, the interval Vague attribute value is made by weighted operation, and the ideal and negative ideal solutions are calculated based on the score function. Then the distance of interval Vague value is defined, as well as the distance between each project and the ideal, and negative ideal solutions. The relative adjacent degree is calculated by TOPSIS method, then the order of the projects is confirmed according to the relative adjacent degree. Finally, a case is used to show the process of the method this paper proposed and the validity of this method is proved.

Keywords: interval Vague set, score function, TOPSIS, multi-attribute decision-making.

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1. Introduction

Decision-making is the process of finding the best option from all of the feasible alternatives. Sometimes, decision-making problems considering several criteria are called multi-criteria decision-making (MCDM) problems. The MCDM problems may be divided into two kinds. One is the classical MCDM problems (Hwang and Yoon 1981; Kaklauskas *et al.* 2006; Zavadskas *et al.* 2008a, b; Lin *et al.* 2008; Ginevičius *et al.* 2008), among which the ratings and the weights of criteria are measured in crisp numbers. Another is the fuzzy multiple criteria decision-making (FMCDM) problems (Bellman and Zadeh 1970; Wang *et al.* 2003; Liu and Wang 2007; Liu and Du 2008; Wei and Liu 2009; Jin *et al.* 2007; Liu and Guan 2008, 2009), among which the ratings and the weights of criteria evaluated on imprecision, subjective

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and vagueness are usually expressed by linguistic terms, fuzzy numbers or intuition fuzzy numbers.

Atanasov (1986, 1989) proposed Intuition fuzzy set theory in 1986. Gau and Buehrer (1993) proposed the concept of Vague set at 1993. Bustince and Burillo (1996) proposed that Vague set was intuition fuzzy set and unified the intuition fuzzy set and the Vague set. As the Vague set (Bustince and Burillo 1996) took the membership degree, non-membership degree and hesitancy degree into account, and has more ability to deal with uncertain information than traditional fuzzy set, lots of scholars pay attentions to the research of Vague set. Atanassov and Gargov (1989) extended the intuition vague set and proposed the concept of interval intuition vague set, also named interval Vague set. Interval Vague set has more ability to express vagueness and uncertainty. At present, the concerned researches of Vague set are focused on the operation rules of the interval Vague set (Atanasov 1994), correlation degree (Bustince and Burillo 1995; Hong 1998), and topological structure (Mondal and Samanta 2001). There are less researches in the area of multi-attribute decision-making. Xu and Chen (2007) proposed interval intuition prefer information operator and hybrid integrated operator, and proposed a jugement method which the prefered information of the decision maker is an interval-valued intuitionistic judgment matrices. But the attributes' weights of this method are general real numbers and have some limitation. Zhou and Wu (2006) proposed a multiple criteria decision method based on interval – value Vague sets of distance. But this method's attributes' weights are also general real numbers. Wang (2006) proposed a multi-criteria decision-making method based on interval-value vague sets with uncertain information. It firstly calculates the common real number attribute weight through evidence theory, then defines the ideal solution and the negative ideal solution, and constructs nonlinear model base on distance. This method is essential for interval intuition decision making with weights of real number type. Multi-attribute interval Vague set decision making methods that the weights are also interval vague value are rare. In this paper, the attribute value and weight are all vague value, a method using TOPSIS method to solve the multi-attribute decision making problem is proposed. Firstly, according to the operation rules of the interval value, the interval vague attribute value has done weighted calculation. And the ideal and negative solutions are confirmed based on the score function. Then the distance of the interval vague value is defined, and the distances between each project and the ideal and negative ideal solutions are calculated. According to the TOPSIS method, the relative adjacent degrees are calculated. Then the order of the projects is confirmed according to the relative adjacent degrees.

2. Evaluation method

2.1. The definition and the essential operations of the interval Vague set

2.1.1. The definition of the interval Vague set

Definition 1 (Atanasov 1986): Suppose discourse domain $X = \{x_1, x_2, ..., x_n\}$, a Vague set A is described by true membership function t_A and false membership function in discourse domain, X, $t_A : X \to [0,1]$, $f_A : X \to [0,1]$. Where, $t_A(x_i)$ is the lower bound that affirms the membership exported by the evidence that support x_i, $f_A(x_i)$ is the lower bound that negates the membership exported by the evidence that support x_i and $t_A(x_i) + f_A(x_i) \leq 1$. The membership of the element x_i in Vague set A is defined by a subinterval $[t_A(x_i), 1 - f_A(x_i)]$ in interval $[0,1]$, and this interval is called the Vague value of x_i in set A. To Vague set A, the representation forms are as follows:

$$A = \begin{cases} \sum_{i=1}^{n} \left[t_A(x_i), 1 - f_A(x_i)\right] / x_i \quad , x_i \in X \quad \text{when } X \text{ is discrete,} \\ \int_X \left[t_A(x), 1 - f_A(x)\right] / x \quad , x \in X \quad \text{when } X \text{ is continuous.} \end{cases} \tag{1}$$

$\forall x \in X$, $\pi_A(x) = 1 - t_A(x) - f_A(x)$ is called Vague degree of x compared with Vague set A. $\pi_A(x)$ shows the hesitation degree or uncertain degree. Obviously, $0 \leq \pi_A(x) \leq 1, x \in X$.

Because of the uncertainty and complexity of the decision, the values of $t_A(x)$ and $f_A(x)$ are difficult to express by accurate real numbers value. The interval values are more flexible than the real number values, extending $t_A(x)$ and $f_A(x)$ from real number value to interval value intuition set can get the interval Vague set. Obviously, this set is much stronger to show the uncertain data and intuition data. The interval Vague value is denoted as $\tilde{x} = <\tilde{t}_x, \tilde{f}_x>$, where $\tilde{t}_x = [t_x^-, t_x^+] \subseteq [0,1]$, $\tilde{f}_x = [f_x^-, f_x^+] \subseteq [0,1]$, and the following equation is satisfied: $t_x^+ + f_x^+ \leq 1$.

$\tilde{\pi}_A(x) = [1,1] - \tilde{t}_A(x) - \tilde{f}_A(x) = [1 - t^+{}_A(x) - f^+{}_A(x), 1 - t^-{}_A(x) - f^-{}_A(x)]$ is called the hesitancy degree of the interval Vague value.

2.1.2. The essential operations of the interval Vague set (Gau and Buehrer 1993, Xu and Chen 2007, Li and Rao 2001)

Suppose there is an interval Vague value $\tilde{x} = <\tilde{t}_x, \tilde{f}_x> = <[t_x^-, t_x^+], [f_x^-, f_x^+]>$, $\tilde{y} = <\tilde{t}_y, \tilde{f}_y> = <[t_y^-, t_y^+], [f_y^-, f_y^+]>$, where $\tilde{t}_x, \tilde{f}_x, \tilde{t}_y, \tilde{f}_y \subseteq [0,1]$ and $t_x^+ + f_x^+ \leq 1, t_y^+ + f_y^+ \leq 1$. The following operation rules and relations can be received:

$$\bar{\tilde{x}} = <\tilde{f}_x, \tilde{t}_x> = <[f_x^-, f_x^+], [t_x^-, t_x^+]>, \tag{2}$$

$$\tilde{x} + \tilde{y} = <\tilde{t}_x + \tilde{t}_y - \tilde{t}_x\tilde{t}_y, \tilde{f}_x\tilde{f}_y> = \\ <[t_x^- + t_y^- - t_x^-t_y^-, t_x^+ + t_y^+ - t_x^+t_y^+], [f_x^- f_y^-, f_x^+ f_y^+]>, \tag{3}$$

$$\tilde{x} \times \tilde{y} = <\tilde{t}_x\tilde{t}_y, \tilde{f}_x + \tilde{f}_y - \tilde{f}_x\tilde{f}_y> = \\ <[t_x^-t_y^-, t_x^+t_y^+], [f_x^- + f_y^- - f_x^- f_y^-, f_x^+ + f_y^+ - f_x^+ f_y^+]>, \tag{4}$$

$$\lambda \times \tilde{x} = <[1 - (1 - t_x^-)^\lambda, 1 - (1 - t_x^+)^\lambda], [(f_x^-)^\lambda, (f_x^+)^\lambda]> \quad \lambda \geq 0. \tag{5}$$

The results of the operation are all interval vague value. According to the operation rules, the following relations can be received:

(1) $\tilde{x}+\tilde{y}=\tilde{y}+\tilde{x}$,

(2) $\tilde{x}\times\tilde{y}=\tilde{y}\times\tilde{x}$,

(3) $\lambda(\tilde{x}+\tilde{y})=\lambda\tilde{x}+\lambda\tilde{y} \quad \lambda\geq 0$,

(4) $\lambda_1\tilde{x}+\lambda_2\tilde{x}=(\lambda_1+\lambda_2)\tilde{x} \quad \lambda_1,\lambda_2\geq 0$.

2.1.3. The vague distances the interval Vague values

Suppose: X is a discourse domain of n elements, A and B are Vague sets, and

$$A=\left\{<[t_A^-(x),t_A^+(x)],[f_A^-(x),f_A^+(x)]>|\, x\in X\right\}.$$

$$B=\left\{<[t_B^-(x),t_B^+(x)],[f_B^-(x),f_B^+(x)]>|\, x\in X\right\}.$$

then the distance between Vague sets A and B has different definition, one of the definition in literature (Zhou and Wu 2006) is shown as follows:

$$d(A,B)=\frac{1}{n}\sum_{i=1}^{n}\max(|t_A^-(x_i)-t_B^-(x_i)|,|t_A^+(x_i)-t_B^+(x_i)|,\; |f_A^-(x_i)-f_B^-(x_i)|,|f_A^+(x_i)-f_B^+(x_i)|). \quad (6)$$

The definition in literature (Wang, 2006) is shown as follows:

$$d(A,B)=\frac{1}{4n}\sum_{i=1}^{n}(|t_A^-(x_i)-t_B^-(x_i)|+|t_A^+(x_i)-t_B^+(x_i)|+|f_A^-(x_i)-f_B^-(x_i)|+ \quad (7)$$
$$|f_A^+(x_i)-f_B^+(x_i)|+|\pi_A^-(x_i)-\pi_B^-(x_i)|+|\pi_A^+(x_i)-\pi_B^+(x_i)|),$$

where

$$\pi_A^-(x)=1-t_A^+(x)-f_A^+(x),\tilde{\pi}_A^+(x)=1-t_A^-(x)-f_A^-(x),$$

$$\pi_B^-(x)=1-t_B^+(x)-f_B^+(x),\tilde{\pi}_B^+(x)=1-t_B^-(x)-f_B^-(x).$$

Obviously, the definition in equation (6) is lock of the signification of distance. Otherwise, it only adopts the interval vague value to get a part of information. So this paper adopts the equation (7) to define the distance of Vague set A and B.

2.2. The description of decision problems based on interval Vague set

Definition 3: there is a multi-attribute decision making problem, suppose $A=\{A_1, A_2, ..., A_m\}$ is a decision project set, $C=\{C_1, C_2, ..., C_n\}$ is the attribute set of the projects. Suppose the character of decision project A_i to attribute set C_j is denoted by interval Vague set: $\tilde{\phi}_{ij}=<\tilde{t}_{ij},\tilde{f}_{ij}>$, where, $\tilde{t}_{ij}=[t_{ij}^-,t_{ij}^+]\subseteq[0,1]$ denotes the degree that decision project A_i

satisfies attribute C_j. $\tilde{f}_{ij} = [f_{ij}^-, f_{ij}^+] \subseteq [0,1]$ denotes the degree that the decision project A_i does not satisfy attribute C_j and the follows is satisfied: $t_{ij}^+ + f_{ij}^+ \le 1$. Suppose, the attribute weight is $W = (\tilde{w}_1, \tilde{w}_2, \cdots, \tilde{w}_n)$, where, $\tilde{w}_j$ is denoted by the interval Vague value as $< \tilde{t}_{w_j}, \tilde{f}_{w_j} >$, and $\tilde{t}_{w_j} = [t_{w_j}^-, t_{w_j}^+] \subseteq [0,1]$, $\tilde{f}_{w_j} = [f_{w_j}^-, f_{w_j}^+] \subseteq [0,1], t_{w_j}^+ + f_{w_j}^+ \le 1$. Using the above attribute weights and interval vague value of different attributes to determine the order of the projects.

2.3. The score function

In order to compare the interval vague value, the score function of the interval Vague value is shown as follows. The interval value vague set is the extension of real number vague. So the real number Vague set score function in existence is extended appropriately in order to construct the interval Vague set score function.

Suppose there is a Vague value $< [t_x^-, t_x^+], [f_x^-, f_x^+] >$, according to the definition of the score function S of Chen and Tan (1994) and the characteristic of the interval number, the score function of the interval Vague value is defined as follows:

$$S_{ij} = \frac{t_{ij}^- + t_{ij}^+}{2} - \frac{f_{ij}^- + f_{ij}^+}{2}. \tag{8}$$

Compare the score function S value, determine the interval Vague value. The bigger the value of the score function S, the bigger the corresponding interval Vague value. But when the value of the score function is equivalent and the number is more than two, this method cannot do the judgment.

Hong and Choi (2000) analysed the deficiency of the score function, and added the precise function H. According to the characteristics of interval vague value, the precise function is defined as follows:

$$H_{ij} = \frac{t_{ij}^- + t_{ij}^+}{2} + \frac{f_{ij}^- + f_{ij}^+}{2}. \tag{9}$$

It shows the precision of the membership situation that is reflected by the interval Vague set. When the values of the score function S are the same, the precise function H is compared. If H is bigger, the corresponding interval vague value is also bigger.

Using an example Liu (2004) prove that the decision making can be done more reasonably through analyzing the crowd that drop their rights. The thought of this method is shown as follows: through analyzing the hesitancy degree $\tilde{\pi}_{ij}$ of the decision making, divide it into three parts according to the result of the voting model: $\tilde{t}_{ij}\tilde{\pi}_{ij}, \tilde{f}_{ij}\tilde{\pi}_{ij}, \tilde{\pi}_{ij}^2$, they denote the ratio of the amount of the supporter to that of the objector and that of the waiver. So, the support degree is $\tilde{t}_{ij} + \tilde{t}_{ij}\tilde{\pi}_{ij}$, and the object degree is $\tilde{f}_{ij} + \tilde{f}_{ij}\tilde{\pi}_{ij}$. $t_{ij}^* = \frac{t_{ij}^- + t_{ij}^+}{2}$, $f_{ij}^* = \frac{f_{ij}^- + f_{ij}^+}{2}$, $\pi_{ij}^* = \frac{\pi_{ij}^- + \pi_{ij}^+}{2}$. Then the definition of the modified score function S1 is as follows:

$$S1_{ij} = t_{ij}^{*} + t_{ij}^{*}\pi_{ij}^{*} - (f_{ij}^{*} + f_{ij}^{*}\pi_{ij}^{*}) = (t_{ij}^{*} - f_{ij}^{*})(1+\pi_{ij}^{*}) . \quad (10)$$

When the values of the score function S are the same and the precise function H are also the same, we use S1to confirm the interval vague value. The bigger S1 is, the bigger is the corresponding interval vague value.

2.4. The processes of the multi-attribute decision making based on the interval Vague set and the TOPSIS

TOPSIS is used to confirm the order of the evaluation objects in virtue of the ideal solution and the negative ideal solution of the multi-attribute problems (Yue 2003). The ideal solution is a best solution that is assumed (marked as V^+). Each of its indicator value is the best value of the optional schemes. The negative solution is another worst solution that is assumed (marked as V^-). Each of its indicator value is the worst value of the optional project. V^+ and V^- are compared with each project in the original project set. The distance information of them is used to be the standard to confirm the order of the projects in X.

(1) Weight the attribute value of each project

According to Eq.(4), calculate the following value:

$$\tilde{b}_{ij} = \tilde{w}_j \tilde{\phi}_{ij} = < \tilde{t}_{b_{ij}}, \tilde{f}_{b_{ij}} > = < [t_{b_{ij}}^{-}, t_{b_{ij}}^{+}], [f_{b_{ij}}^{-}, f_{b_{ij}}^{+}] > , \quad (11)$$

where,

$$\tilde{t}_{b_{ij}} = [t_{ij}^{-} t_{w_j}^{-}, t_{ij}^{+} t_{w_j}^{+}] , \quad (12)$$

$$\tilde{f}_{b_{ij}} = [f_{ij}^{-} + f_{w_j}^{-} - f_{ij}^{-} f_{w_j}^{-}, f_{ij}^{+} + f_{w_j}^{+} - f_{ij}^{+} f_{w_j}^{+}] . \quad (13)$$

(2) Calculate the score function

According to Eq.(8), Eq.(9) and Eq.(10), calculate the score function, precise function and corrected function of $\tilde{b}_{ij}$ separately.

(3) Confirm the ideal solution and the negative solution of the evaluation object

The interval intuitionistic fuzzy set ideal solution V^+ and negative ideal solution V^- is shown as follows:

$$\max\_i = \max_{i}(S_{ij}) , \quad (14)$$

$$V_j^{+} = < [t_{V_j^+}^{-}, t_{V_j^+}^{+}], [f_{V_j^+}^{-}, f_{V_j^+}^{+}] > = < [t_{b_{\max\_ij}}^{-}, t_{b_{\max\_ij}}^{+}], [f_{b_{\max\_ij}}^{-}, f_{b_{\max\_ij}}^{+}] > , \quad (15)$$

$$\min\_i = \min_{i}(S_{ij}) , \quad (16)$$

$$V_j^{-} = < [t_{V_j^-}^{-}, t_{V_j^-}^{+}], [f_{V_j^-}^{-}, f_{V_j^-}^{+}]] > = < [t_{b_{\min\_ij}}^{-}, t_{b_{\min\_ij}}^{+}], [f_{b_{\min\_ij}}^{-}, f_{b_{\min\_ij}}^{+}] > . \quad (17)$$

(4) Calculate the distance between each project and the ideal solution

According to Eq.(7), the distance between each project and the ideal solution is calculated as follows:

$$d_i^+ = \frac{1}{4n}\sum_{j=1}^{n}(|t_{b_{ij}}^- - t_{V_j^+}^-| + |t_{b_{ij}}^+ - t_{V_j^+}^+| + |f_{b_{ij}}^- - f_{V_j^+}^-| + |f_{b_{ij}}^+ - f_{V_j^+}^+| + |\pi_{b_{ij}}^- - \pi_{V_j^+}^-| + |\pi_{b_{ij}}^+ - \pi_{V_j^+}^+|). \quad (18)$$

(5) Calculate the distance between each project and the negative ideal solution

According to Eq.(7), the distance between each project and the negative ideal solution is calculated as follows:

$$d_i^- = \frac{1}{4n}\sum_{j=1}^{n}(|t_{b_{ij}}^- - t_{V_j^-}^-| + |t_{b_{ij}}^+ - t_{V_j^-}^+| + |f_{b_{ij}}^- - f_{V_j^-}^-| + |f_{b_{ij}}^+ - f_{V_j^-}^+| + |\pi_{b_{ij}}^- - \pi_{V_j^-}^-| + |\pi_{b_{ij}}^+ - \pi_{V_j^-}^+|). \quad (19)$$

(6) Confirm the relative adjacent degree

The relative adjacent degree of the evaluation object and the ideal solution is:

$$C_i = \frac{d_i^-}{d_i^+ + d_i^-}.(i = 1, 2, \cdots, m)\cdot \quad (20)$$

According to the relative adjacent degree, the order of the evaluation objects can be confirmed and the best appropriate object can be selected.

3. Application example

A company intends to select a person to take the department manager position. Four aspects of the candidate are evaluated by the experts. The four aspects are ideological and moral quality (C_1), professional ability (C_2), creative ability (C_3) and knowledge range (C_4). The experts give evaluation data and weights to each aspects. And they are all denoted by the interval Vague value, namely, the interval number of the support degree is given, and the interval number of the object degree is also given. The evaluation data and attribute weight is shown by Tables 1 and 2. The order of the 3 candidates must be confirmed.

Table 1. The evaluation data of a different candidate given by the experts

	C_1 C_2 C_3 C_4
S1	([0.6,0.7],[0.2,0.3]) ([0.5,0.6],[0.1,0.3]) ([0.6,0.7],[0.2,0.3]) ([0.3,0.4],[0.5,0.6])
S2	([0.4,0.5],[0.3,0.4]) ([0.7,0.8],[0.1,0.2]) ([0.5,0.6],[0.3,0.4]) ([0.4,0.5],[0.3,0.4])
S3	([0.5,0.6],[0.3,0.5]) ([0.3,0.4],[0.3,0.5]) ([0.6,0.7],[0.1,0.3]) ([0.5,0.6],[0.1,0.3])

Table 2. The attribute weight given by the experts

	C_1	C_2	C_3	C_4
W	([0.3,0.4],[0.5,0.6])	[0.6,0.7],[0.1,0.2])	([0.6,0.7],[0.2,0.3])	([0.3,0.4],[0.5,0.6])

The process to confirm the order is shown as follows:

(1) Calculate the weighted b_{ij}

$$b = \begin{bmatrix} ([0.18,0.28],[0.60,0.72]) & ([0.30,0.42],[0.19,0.44]) & ([0.36,0.49],[0.36,0.51]) & ([0.09,0.16],[0.75,0.84]) \\ ([0.12,0.20],[0.65,0.76]) & ([0.42,0.56],[0.19,0.36]) & ([0.30,0.42],[0.44,0.58]) & ([0.12,0.20],[0.65,0.76]) \\ ([0.15,0.24],[0.65,0.80]) & ([0.18,0.28],[0.37,0.60]) & ([0.36,0.49],[0.28,0.51]) & ([0.15,0.24],[0.55,0.72]) \end{bmatrix}$$

(2) The score function is calculated as follows

$$S = \begin{bmatrix} -0.430 & 0.045 & -0.010 & -0.670 \\ -0.545 & 0.215 & -0.150 & -0.545 \\ -0.530 & -0.255 & 0.030 & -0.440 \end{bmatrix}$$

$$H = \begin{bmatrix} 0.890 & 0.675 & 0.860 & 0.920 \\ 0.865 & 0.765 & 0.870 & 0.865 \\ 0.920 & 0.715 & 0.820 & 0.830 \end{bmatrix}$$

(3) Confirm the ideal solution and the negative solution

All the indicators are benefit type. The ideal solution and the negative solution are confirmed as follows:

$$V^{+} = \{([0.18,0.28],[0.6,0.72])\ ([0.42,0.56],[0.19,0.36])\ ([0.36,0.49],[0.28,0.51])\ ([0.15,0.24],[0.55,0.72])\}.$$

$$V^{-} = \{([0.12,0.2],[0.65,0.76])\ ([0.18,0.28],[0.37,0.6])\ ([0.3,0.42],[0.44,0.58])\ ([0.09,0.16],[0.75,0.84])\}.$$

(4) Calculate the distance between each project and the negative solution

$$d^{+} = (0.0825, 0.06375, 0.08125),$$

$$d^{-} = (0.07875, 0.0875, 0.0825).$$

(5) Calculate the relative adjacent degree:

$$C = (0.4884, 0.5785, 0.5038).$$

(6) Confirm the order of the candidates

According to the relative adjacent degree, the order of 3 candidates can be confirmed:

$$S_2 \succ S_3 \succ S_1.$$

So the second candidate S_2 is the best one.

4. Conclusion

As the decision making is fuzzy and uncertain, the interval Vague set's ability to express the fuzziness and uncertainty is stronger. It is easier to express the decision information using the interval vague value. This paper explored the multi-attribute decision making problem based on the interval vague value. Firstly, according to the operation rules of the interval vague value, do weighted operation to the interval Vague attribute value. And the ideal and negative ideal solutions are confirmed on the basis of the score function. Then the distance of the interval vague value is defined and the distances between each project and the ideal and negative ideal solutions are calculated. The relative adjacent degree is calculated by the TOPSIS method. According to the relative adjacent degree, the order of the project is confirmed. This paper offers a new efficient approach to deal with the multi-attribute decision making problems based on the interval vague value. The theory of the interval Vague set is enriched and developed, and the traditional TOPSIS method is extended and extends its application bound. The other application of the interval vague set is waiting to be researched and explored.

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DAUGIAKRITERINĖS SPRENDIMO PRIĖMIMO PROBLEMOS SPRENDIMAS TAIKANT NEAPIBRĖŽTĄ INTERVALĄ IR TOPSIS METODĄ

P. Liu

Santrauka

Straipsnyje siūlomas daugiakriterinės sprendimo priėmimo problemos sprendimas TOPSIS metodu, kai kriterijų reikšmingumai ir reikšmės yra intervaliniai dydžiai. Iš pradžių, naudojantis procedūromis, nustatomos svertinės intervalinių dydžių reikšmės, paskui apskaičiuojami idealiai teigiamas ir idealiai negiamas sprendiniai. Toliau nustatomi intervalų dydžiai, apskaičiuojami atstumai tarp kiekvienos alternatyvos ir idealiai teigiamo ir idealiai neigiamo sprendinių. TOPSIS metodu apskaičiuojami santykiniai atstumai iki minėtų idealių sprendinių ir alternatyvos išrikuojamos į eilę. Galiausiai konkrečiu pavyzdžiu demonstruojamas skaičiavimo procesas ir patvirtinamas siūlomo metodo pagrįstumas.

Reikšminiai žodžiai: intervalinės neapibrėžtos reikšmės, funkcija, TOPSIS, daugiakriterinis sprendimų priėmimas.

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